

Absolute Alpha by Beta Manipulations

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Abstract

This paper describes a method of achieving an absolute positive alpha by manipulating beta. Instead of creating a new portfolio, an investor can directly invest in market with leverage and he is able to maintain a positive alpha by adjusting beta according to market moving averages. To evaluate this method, the paper presents empirical evidences of the change of average daily alpha in regard of the change of leverage and different time units of moving average. The results show that high average daily alphas come with high leverage and more days of moving average adjusted when controlling beta.

1 Introduction

In this paper we present a method to achieve an absolute positive alpha by beta manipulation. Scholars as well as investment advisors have spent a lot of energy trying to invent portfolios that can achieve positive returns than market, yet no proven methods have been discovered. Instead of creating a new portfolio, one can simply invest in market with risks adjusted depending on market returns and he is able to maintain a positive alpha consistently. John Bogle discussed in his book *The Clash of the Cultures: Investment vs. Speculation* that the size of all available index funds is getting bigger in the last several decades [1]. This model presented in this paper serves as a support so that investors can build up more conviction when they are told to invest in the market.

An investor can directly invest in market. The adjustment of risk should be proportional to price-to-moving average ratio with leverage. Intuitively speaking, an investor should be heavy on leverage when market generates non-negative returns and should be light or short on leverage when market generates negative returns. We takes the daily prices and returns from 1993 to 2015 and calculates moving averages (Yin, 2013) [2]. Section (2) of this paper will explain the mathematics model.

It is known long moving average of real earnings helps to forecast future real dividends (Cambell

and Shiller, 1988) which is then correlated with returns on stocks [3]. This also points out that long term moving average ignores short term volatility and generates macro perspective business cycles in general economy. In Section (3) of this paper, we present results with different moving averages and leverage level in constructing portfolio. We also present a measurement to compare the how the results change by changing different variables in the model.

2 Mathematical Model

This section will explain the mathematics model and prove the absolute positive alpha under the definitions. We define Simple Moving Average by taking the sum of prices at any point (P_i) and divide the sum by the number of observations (n), namely SMA (Yin, 2013).

$$SMA_n = \frac{1}{n} \sum_{i=1}^N P_T \quad (1)$$

Fama (1968) mentioned Sharpe's insight of equilibrium condition and by taking derivatives he obtained the following equation [4].

$$E(R_i) = R_F + \left[\frac{E(R_M) - R_F}{\sigma^2(R_M)} \right] \times cov(R_i, R_M), \quad i = 1, 2, \dots, N.$$

Assuming there is a difference between access return (expected return of portfolio and risk-free return) and market premium (access return of market comparing to risk-free return). We define this difference to be alpha (α). Hence we rewrite the equation as the following.

$$r_p = r_f + \beta(r_m - r_f) + \alpha \quad (2)$$

An investor has certain amount of buying power and he can also invest heavier than his current buying power allows him to by taking a leverage. Assume he is free to add money in portfolio or liquidate

positions any time he wants. He can directly invest in the market and manipulate β to be proportional to the price-to-moving average ratio or inverse price-to-moving average ratio.

$$\beta = \begin{cases} \frac{P_t}{SMA_n} = \frac{P_m}{SMA_n}, & \text{if } P_m \geq SMA_n \\ \frac{SMA_n}{P_t} = \frac{SMA_n}{P_m}, & \text{if } P_m < SMA_n \end{cases} \quad (3)$$

Next, we can describe the return of his portfolio (r_p) to be the return of market (r_m) times β multiplied by leverage (l). We can write this relation as the following equation.

$$r_p = \begin{cases} r_m \times \beta \times l, & \text{if } r_m \geq 0 \\ r_m \times (-\beta) \times l, & \text{if } r_m < 0 \end{cases} \text{ while } l \in (1, \infty) \quad (4)$$

This relation tells us that the return of this investor's portfolio is tied up to the market with a certain leverage based on the sign of market return. When market returns positive, it makes sense for an investor to be heavy on market. The relationship between market return and portfolio return drops when market is coming down and is to cross over the moving average. This is the time when the investor should liquidate assets and invest in assets that negatively correlated to market. He can achieve this goal by buying VXX or short futures. Disregard the approaches, there is no reason for an investor to hold the market when this investment is losing money.

Under the definition of equation (3), beta is always positive and always larger than one. Leverage (l) is always larger than one as well. An investor can certain amount of money that he is willing to put in stock market. He is allowed to invest a portion of that money. He is also allowed to invest all of that money. Furthermore, he can borrow money to increase his buying power. When we say leverage, we only refer to the last scenario where an investor is borrowing money to invest. Hence, the buying power always exceeds the maximum amount of money this investor started with. That is to say, leverage (l) is always larger than one.

Under the definitions above this investor can maintain a positive alpha by adjusting leverage and weight of his portfolio. We call this the Absolute Alpha Theory (since alpha is an absolute value, i.e. a

non-negative value).

Absolute Alpha Theory: An investor can achieve an absolute non-negative alpha by investing directly in market with beta adjusted to price-to-moving average ratio and by divesting or shorting the market with beta adjusted to negative moving average-to-price ratio.

Proof: Assume $r_m \geq 0$, we start off by writing down Capital Asset Pricing Model. We derived the following equation from Sharpe (1964) capital asset prices model (see Appendix 6.1).

$$r_p = r_f + \beta(r_m - r_f) + \alpha$$

Substitute r_p from equation (4) with β from equation (3) plugged into equation (4):

$$\frac{P_m}{SMA_n} l \times r_m = r_f + \beta(r_m - r_f) + \alpha$$

Rewrite equation with α on the left hand side:

$$\alpha = \frac{P_m}{SMA_n} l \times r_m - r_f - \beta(r_m - r_f) = \left(\frac{P_m}{SMA_n} l - \beta \right) \times r_m - r_f + \beta \times r_f$$

Since $r_f \approx 0$ so let $r_f = 0$, then

$$\alpha = \left(\frac{P_m}{SMA_n} l - \beta \right) \times r_m = \frac{P_m}{SMA_n} (l - 1) \times r_m \quad (5)$$

Since $r_m \geq 0$ and $l > 1$, each of the terms in equation (5) is positive. This gives us

$$\alpha \geq 0$$

Assume $r_m < 0$, we start off by writing down Capital Asset Pricing Model.

$$r_p = r_f + \beta(r_m - r_f) + \alpha$$

Substitute r_p from equation (4) with β from equation (3) plugged into equation (4):

$$-\frac{SMA_n}{P_m} l \times r_m = r_f + \beta(r_m - r_f) + \alpha$$

Rewrite equation with α on the left hand side:

$$\alpha = -\frac{SMA_n}{P_m}l \times r_m - r_f - \beta(r_m - r_f) = (-\frac{SMA_n}{P_m}l - \beta) \times r_m - r_f + \beta \times r_f$$

Since $r_f \approx 0$ so let $r_f = 0$, then

$$\alpha = (-\frac{SMA_n}{P_m}l - \beta) \times r_m = -\frac{SMA_n}{P_m}(l + 1) \times r_m \quad (6)$$

Since $r_m < 0$ and $l > 1$, negative signs cancel each other in equation (6) and this leaves the final result to be

$$\alpha > 0$$

Q.E.D.

The mathematics model also relies on the number of days used in calculating the moving averages. A moving average with less days tend to be more volatile than a moving average with more days. On the other side, a moving average with more days give you a smaller value than that with less days and hence require bigger buying power. However, bigger buying power also implies higher level of difficulty when facing less liquidity stocks. The next section will take this factor into account and evaluate the mathematics model.

3 Data and Results

This section will present the results of the empirical evidence and the measurement of average daily alpha. We collected daily price and calculated daily return of S&P 500 from 1993 to 2015. We know that both moving average and leverage can affect the result of alpha. We chose different days of moving averages (i.e. 10-day, 20-day, 30-day, 40-day, 50-day, 100-day, 200-day, 300-day, 400-day, 500-day) in the experiment. We also chose different leverage (from 1 to 30, whole numbers).

We evaluate the model in Section (2) by calculating the average daily alpha. We have total N observations and we have total n days of moving average. We calculate the sum of all alphas and divide the

sum by $(N-n)$ to obtain the average daily alpha ($\bar{\alpha}$).

We take equation (5) and calculate the average daily α with N observations when market return is non-negative:

$$\bar{\alpha}_{r_m \geq 0} = \frac{1}{N-n} \sum_{n=1}^{\infty} \alpha = \frac{1}{N-n} \sum_{n=1}^{\infty} \frac{P_m}{SMA_n} (l-1) \times r_m \quad (7)$$

We take equation (6) and calculate the average daily α with N observations when market return is negative:

$$\bar{\alpha}_{r_m < 0} = \frac{1}{N-n} \sum_{n=1}^{\infty} \alpha = \frac{1}{N-n} \sum_{n=1}^{\infty} \frac{SMA_n}{P_m} (l-1) \times r_m \quad (8)$$

We separate equation (7) and (8) because we have different market return condition. We then calculate average daily alpha by Simple Moving Average in number of days (SMA_n) and leverage (l). We have two inputs controlled and we have the following table with average daily alpha ($\bar{\alpha}$) calculated in percentage (%). In Figure (1), we observe that the average daily alpha ($\bar{\alpha}$) increases when Simple Moving Average (SMA_n) and leverage (l) increase. If we control leverage ($l = 1$), then we observe that daily average alpha ($\bar{\alpha}$) increases when Simple Moving Average increases. If we control Simple Moving Average (SMA_n while $n = 10$), then we observe that daily average alpha ($\bar{\alpha}$) increases when leverage (l) increases.

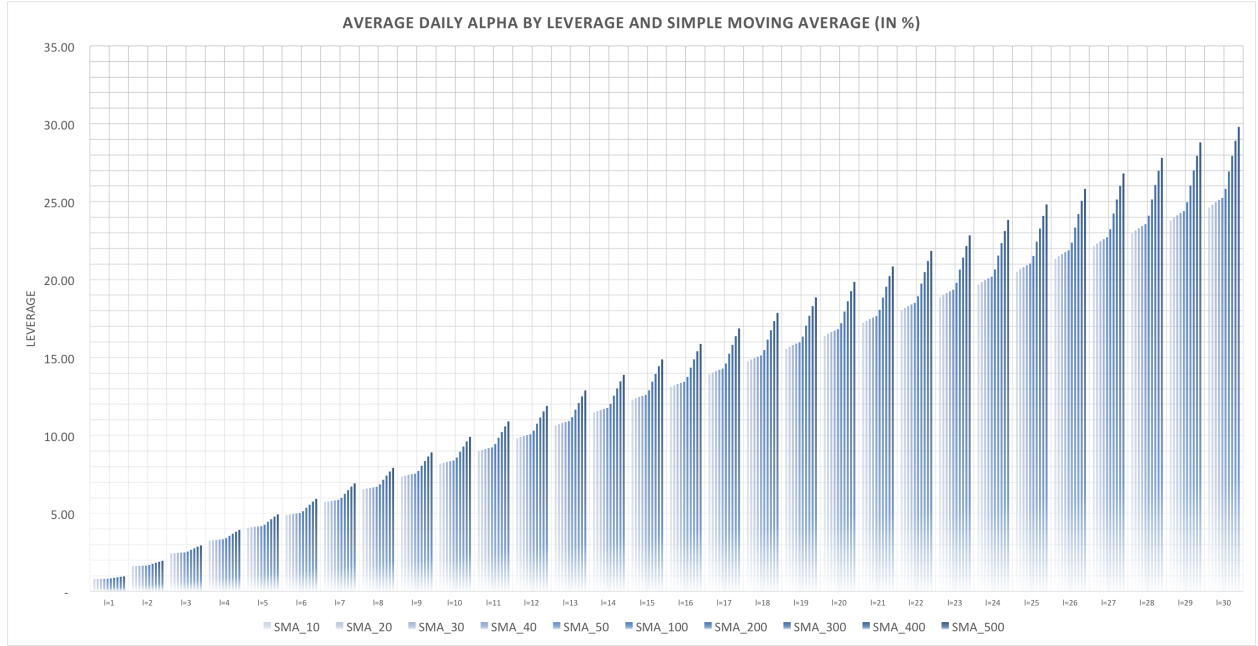
From year 1993, an investor can achieve a daily average alpha ($\bar{\alpha}$) to be 0.79% if he chooses to invest all of his money ($l = 1$, i.e. not a penny more or less) into the market adjusting beta by using 10-day moving average. This would generate higher frequency of trading activities. However, he could do a lot better to increase his daily average alpha ($\bar{\alpha}$) to 0.96% ($\approx 1\%$) if he was looking at a moving average regarding more days into the past, say 500-day. He will be adjusting his portfolio less frequently than looking at less days and it will cost him less (since getting in and out of a position charges commission fee). He could also do better if he is willing to take on a leverage.

Figure (1). *Daily prices and returns collected from 1993 to 2015. Average daily alpha ($\bar{\alpha}$) calculated in percentage (%). The row on top presents Simple Moving Average (SMA_n) with ten selected sample days (10-day, 20-day, 30-day, 40-day, 50-day, 100-day, 200-day, 300-day, 400-day, 400-day). The column on the left presents Leverage (l) with thirty selected sample whole numbers (1 to 30).*

Ave(a)	SMA_n (in %)									
Leverage	SMA_{10}	SMA_{20}	SMA_{30}	SMA_{40}	SMA_{50}	SMA_{100}	SMA_{200}	SMA_{300}	SMA_{400}	SMA_{500}
l=1	0.79	0.79	0.80	0.80	0.81	0.83	0.87	0.90	0.93	0.96
l=2	1.61	1.62	1.63	1.64	1.65	1.69	1.76	1.83	1.90	1.95
l=3	2.43	2.45	2.47	2.48	2.49	2.55	2.66	2.76	2.86	2.95
l=4	3.25	3.28	3.30	3.32	3.34	3.41	3.56	3.69	3.82	3.94
l=5	4.07	4.11	4.13	4.16	4.18	4.28	4.46	4.63	4.79	4.94
l=6	4.90	4.93	4.97	5.00	5.02	5.14	5.36	5.56	5.75	5.93
l=7	5.72	5.76	5.80	5.83	5.86	6.00	6.26	6.49	6.72	6.93
l=8	6.54	6.59	6.63	6.67	6.71	6.86	7.16	7.42	7.68	7.92
l=9	7.36	7.42	7.47	7.51	7.55	7.72	8.06	8.36	8.65	8.91
l=10	8.18	8.25	8.30	8.35	8.39	8.59	8.96	9.29	9.61	9.91
l=11	9.00	9.08	9.13	9.19	9.23	9.45	9.86	10.22	10.58	10.90

l=12	9.83	9.90	9.97	10.02	10.08	10.31	10.75	11.15	11.54	11.90
l=13	10.65	10.73	10.80	10.86	10.92	11.17	11.65	12.09	12.51	12.89
l=14	11.47	11.56	11.63	11.70	11.76	12.03	12.55	13.02	13.47	13.89
l=15	12.29	12.39	12.47	12.54	12.61	12.90	13.45	13.95	14.44	14.88
l=16	13.11	13.22	13.30	13.38	13.45	13.76	14.35	14.88	15.40	15.88
l=17	13.93	14.04	14.13	14.22	14.29	14.62	15.25	15.82	16.37	16.87
l=18	14.76	14.87	14.97	15.05	15.13	15.48	16.15	16.75	17.33	17.86
l=19	15.58	15.70	15.80	15.89	15.98	16.34	17.05	17.68	18.30	18.86
l=20	16.40	16.53	16.63	16.73	16.82	17.21	17.95	18.61	19.26	19.85
l=21	17.22	17.36	17.47	17.57	17.66	18.07	18.85	19.55	20.23	20.85
l=22	18.04	18.19	18.30	18.41	18.50	18.93	19.74	20.48	21.19	21.84
l=23	18.86	19.01	19.13	19.24	19.35	19.79	20.64	21.41	22.16	22.84
l=24	19.69	19.84	19.97	20.08	20.19	20.65	21.54	22.34	23.12	23.83
l=25	20.51	20.67	20.80	20.92	21.03	21.51	22.44	23.28	24.09	24.83
l=26	21.33	21.50	21.63	21.76	21.87	22.38	23.34	24.21	25.05	25.82
l=27	22.15	22.33	22.47	22.60	22.72	23.24	24.24	25.14	26.02	26.81
l=28	22.97	23.15	23.30	23.44	23.56	24.10	25.14	26.07	26.98	27.81
l=29	23.79	23.98	24.13	24.27	24.40	24.96	26.04	27.01	27.95	28.80
l=30	24.62	24.81	24.97	25.11	25.24	25.82	26.94	27.94	28.91	29.80

Figure (2). *This chart plots all the observations from Figure (1). For each unit of Simple Moving Average (SMA_n) as x-axis, the color of the column increases from light to dark, symbolizing 10-day to 500-day. For y-axis, leverage (l) is presented from 1 to 30.*



4 Conclusion

This paper introduces a method to achieve an absolute non-negative alpha consistently by adjusting beta to proportionate to price-to-moving average ratio. We further prove the Absolute Alpha Theory. We also introduce average daily alpha to measure the validity of the mathematics model. We further present empirical evidence and illustrate the results.

The results illustrate the proof of Absolute Alpha Theory. However, such method may or may not be easy to execute in real life situation. Depending on the buying power and leverage size, the portfolio could get into liquidity problem. Depending on the volatility and the time frame of the market, an investor could be required to adjust portfolio for multiple trading days in a row since price could cross over moving average back and forth before the trend becomes clear.

Rather to be a ground breaking idea, this paper implies an investor should be humble in front of market. This paper also suggests an investor to commit less frequent trading activities by selecting longer

time frame of investment horizon to achieve higher alpha.

The philosophy of this paper follows value investing and Efficient Market Hypothesis. Although paper introduces an idea to beat the market, the philosophy lands on an advice to invest in market, to trade less, and to hold the portfolio in long-term.

5 Acknowledge

I want to thank Prof. Gregg Jarrell on this paper.

6 Appendix

6.1 Derivation of CAPM

For a combination of two risky assets, we have the following weighted expected return, $E_{Rc} = \alpha E_{Rp} + (1 - \alpha) E_{Ra}$. We can also look at the risk by calculating the standard deviation, $\sigma_{Rc} = \sqrt{\alpha^2 \sigma_{Rp}^2 + (1 - \alpha)^2 \sigma_{Ra}^2 + 2\alpha(1 - \alpha)\sigma_{Rp}\sigma_{Ra}}$. With all the available observations plotted on a xy-axis graph, we are interested in a combination that gives us the most optimal return-to-risk ratio. That is, we need to take derivative when one of the weight is zero ($\alpha = 0$).

We can re-arrange the standard deviation of a combination of g and i, and we have the following. $\sigma = \sqrt{\alpha^2 \sigma_{Ri}^2 + (1 - \alpha)^2 \sigma_{Rg}^2 + 2\alpha(1 - \alpha)\sigma_{Ri}\sigma_{Rg}}$. Then we take derivate at $\alpha = 0$, and we obtain,

$$\frac{d\sigma}{d\alpha} = -\frac{1}{\sigma}[\sigma_{Rg}^2 - r_{ig}\sigma_{Ri}\sigma_{Rg}] \quad (9)$$

Next, we look at any group of point (E_{Rg}, σ_{Rg}) on the line. We are able to plug the number in equation (1). We obtain the following.

$$\frac{r_{ig}\sigma_{Ri}}{\sigma_{Rg}} = -\left[\frac{P}{E_{Rg} - P}\right] + \left[\frac{1}{E_{Rg} - P}\right]E_{Ri} \quad (10)$$

The goal here is to re-write the equation into something that we use almost everyday from equation (10). Here we are interested in the origin of capital asset pricing model? From equation (10), Sharpe (1964) defines, $B_{ig} = \frac{r_{ig}\sigma_{Ri}}{\sigma_{Rg}}$, so we can re-write equation (10). $B_{ig} = \frac{r_{ig}\sigma_{Ri}}{\sigma_{Rg}} = -[\frac{P}{E_{Rg}-P}] + [\frac{1}{E_{Rg}-P}]E_{Ri}$. We are looking at a portfolio by looking at its slope. That is, we are looking at a portfolio in terms of volatility. It gives us a very indirect picture and a vague image of what this portfolio actually looks like. We do the following derivation to make it clear.

First we want an expression with expected return on one side of the formula. From equation (2), we have:

$$B_{ig} + \frac{P}{E_{Rg} - P} = \frac{1}{E_{Rg} - P} E_{Ri}$$

Multiply both equation to get rid of the denominator on the right hand side of the formula. Then we re-write the equation with expected return on the left so that we have a mathematics expression of expected return, as described in equation (11).

$$\begin{aligned} (B_{ig} + \frac{P}{E_{Rg} - P}) \times (\frac{1}{E_{Rg} - P})^{-1} &= E_{Ri} \\ E_{Ri} &= (B_{ig} + \frac{P}{E_{Rg} - P}) \times (\frac{1}{E_{Rg} - P})^{-1} \end{aligned} \quad (11)$$

We still have a chunk of “stuff” on the right hand that we cannot visually interpret. We need to multiply these factors out of the parenthesis and re-adjust the formula into something meaningful. As we do this, we will cancel out the denominator and simply the equation, as in equation (12).

$$\begin{aligned} E_{Ri} &= B_{ig} \times (\frac{1}{E_{Rg} - P})^{-1} + \frac{P}{E_{Rg} - P} \times (\frac{1}{E_{Rg} - P})^{-1} \\ E_{Ri} &= B_{ig} \times (E_{Rg} - P) + P \end{aligned} \quad (12)$$

Based on the paper, Sharpe (1964) defined P to be pure interest rate and B to be the ratio of covariance of two risky assets and one of the risky assets. We re-name interest rate to be r_f and systematic risk from the market volatility to be β . We can re-write the equation as the following, equation (13).

$$E_{Ri} = \beta_{ig} \times (E_{Rg} - r_f) + r_f \quad (13)$$

Now we have traditional Capital Asset Pricing Model! The reason this model is powerful is because it allows investors to interpret a return of a risky asset by looking at risk-free return from treasury bill and the market access return to the risk-free return from treasury bill with systematic risk.

7 Reference

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