How to Understand Future Returns of a Security?

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Abstract

This paper is a revise version of a previous paper written by Yiqiao, which discusses a newly developed model that has the strength to generate accurate results on any security in financial market – Regression Analytical Model, which originates from a combination of econometrics method on momentum hypothesis. We extend this model to Two-factor Hypothesis, which explains the future return of a security. The model originates from the goal of understanding the irrational behavior of investors on the financial market.

1. Background

In the various markets that we have today, one can survive only by outperforming the average. But the market, due to its random nature, has a strong irrational behavior attached to it. Moreover, the past has taught us that irrational big market events can have a great impact on the prices of the stocks. For example, when Chipotle Mexican Grill went IPO in January 2006 at the price $22 per share, the price instantly became $44 per share in the next trading day. A more compelling example is, of course, the financial crisis.

The goal of our model is to take into account the irrational nature of the market. However, the model does not include big events such as mergers and acquisitions.

In order to understand our model, which we call Regression Analytical Model (RAM), we first need to introduce some basic concepts that are well known in the field of financial investment. The first one is called Simple Moving Averages (referred to as SMA in the text). The SMA calculates different averages of the prices of the shares over a certain period of time. Starting from these averages, one can define a function, called Exponential Moving Averages (referred to as EMA in the text). The EMA is an attempt to define an exponential function in this context.

What we did was to expand the SMA and EMA into the so-called Momentum Hypothesis. This is an essential tool for our RAM model and one can think of it as way to describe investors' irregular emotional changes in the financial market. We cannot prove the Momentum Hypothesis mathematically, but we will show some example where it was successfully applied to estimate prices.

In this paper, we separate all of the investor emotions into four different types: 1) rationally bearish, 2) rationally bullish, 3) rationally bearish, and 4) irrationally bullish. These terms are classical and their meaning is straight forwards.

The paper is structured as follows: first we review the notions of SMA and EMA. In section 2, we introduce the Momentum Hypothesis, while in section 3 we define the Momentum Regression Analytical Model. The following sections will describe applications of the model with examples and results.

One last observation needs to be made: the Momentum Regression Analytical Model is a more emotional approach of the Momentum Hypothesis, rather than a complicated model built out of the Linear Regression Model.

1.1 Simple Moving Average (SMA)

Appel defined in [2] the concept of Simple Moving Average, which depends on the number of days an investor strategizes in the investment model. The most widely used ones are SMA_5, SMA_{12}, SMA_{26}, and SMA_{50}. The equation is given as follows:

$$Definition\ 1: SMA_n = \frac{1}{n}\sum_{i=1}^{n} x_i$$

In other words, SMA_n is the average of the price of a certain security over n days, so it gives an estimate of what happened in the past n days.

If one has only one SMA_n, available one can only say something about what happened in those n days, but knowing several SMA_n, for different choices of n, one can extract more information. For example, if one knows SMA_{12} and SMA_{26}, we call the market as acting bullish if the price today is larger than SMA_{12} and SMA_{12} is larger than SMA_{26}. To understand the reason, we should think backwards. If a price is higher than it has been n days before, than, assuming a steady evolution, intuitively one should expect that the price should be higher than the averages of the price over n days. Generally, given a randomly distributed group of observations, a single observation in the group is pulling the average up if it is higher than the average. That is to say, if we have observed that the price is larger than short-period $SMAn_1$ and the short-period SMA_{n1} is larger than long-period SMA_{n2} (for any n_1 smaller than n_2), then we are experiencing a market with bullish momentum, and vice versa, which leads to a known hypothesis [2].

$$Hypothesis\ 1: The\ market\ of\ a\ particular\ security\ is\ experiencing\ bullish\ momentum\ if\ and\ only\ if\ for\ any\ m < n, Price > SMA_m > SMA_n; and\ \frac{\partial Price}{\partial time} > \frac{\partial SMA_m}{\partial time} > \frac{\partial SMA_n}{\partial time}.$$

$$Hypothesis\ 1.1: The\ market\ of\ a\ particular\ security\ is\ experiencing\ bearish\ momentum\ if\ and\ only\ if\ for\ any\ m < n, Price < SMA_m < SMA_n; and\ \frac{\partial Price}{\partial time} < \frac{\partial SMA_m}{\partial time} < \frac{\partial SMA_n}{\partial time}.$$

1.2 Exponential Moving Average (EMA)

The EMA is a function that gives us an even better understanding of the market-moving pattern than the SMA because it not only looks at two different prices, but also uses a special exponential function in its equation. The idea behind it can be understood from the following example: Let us suppose that John's behavior represents the actions of most of investors in the market. He is not too smart, and not too stupid. He makes rational decisions, but he makes mistakes occasionally as well. If he wants to take a look at historical data, he would somehow pay less attention on what happened recently. In other words, if he plans to calculate EMA on a relative longer time period, he would pay less attention to today’s closing price, and he would give a much higher percentage or weight to that historical price he wants to look at. The natural instinct would say that, given a steady evolution, what captures most the behavior is not today's price, but what happened in the more distant past.

The EMA captures this idea and it is defined as follows:

$$Definition\ 2: EMA_n = Price * \frac{2}{1+n} + \left(\frac{1}{n}\sum_{i=1}^{n} Price_i\right) * \left(1 - \frac{2}{1+n}\right)\ for\ any\ n\ \in \mathbb{Z}_+.$$

In words, this equation is weighing a certain SMA_n (for a given n) with a coefficient, which decreases if more number of time units is analyzed. This equation is more accurate at measuring market-moving patterns and it follows the same property as SMA.

$$Hypothesis\ 2:\ The\ market\ is\ experiencing\ bullish\ momentum\ if\ and\ only\ if\ for\ any\ m < n\ and\ m,n\ \in \mathbb{Z}_+, Price > EMA_m > EMA_n;\ and\ \frac{\partial Price}{\partial time} > \frac{\partial EMA_m}{\partial time} > \frac{\partial EMA_n}{\partial time};$$

$$Hypothesis\ 2.1:\ The\ market\ is\ experiencing\ bearishh\ momentum\ if\ and\ only\ if\ for\ any\ m < n\ and\ m,n\ \in \mathbb{Z}_+, Price < EMA_m < EMA_n;\ and\ \frac{\partial Price}{\partial time} < \frac{\partial EMA_m}{\partial time} < \frac{\partial EMA_n}{\partial time};$$

To understand this hypothesis, let us go back to our example of average investor John. If John is holding a security that appears to be bullish today, that means there is another investor out there in the market who is willing to buy the share away from John at a price higher than what John has originally paid for. In this case, there are two prices. The first one is the one with which John purchases the stock. The second one is the one with which the other investor purchases stock from John. In order to make a profit, John needs to buy the stock of a security beforehand, so he can actually sell it later. If we are assigning exponential percentages to each of these two prices, we will have a bigger percentage assigned to the price John has purchased the stock for, and a smaller percentage assigned to the price (not including average) that John has sold it for to the other investor.

Now imagine we have a second trade after John has sold the stock, and we assign the probability exponentially to the price. It is obvious that if the price keeps going up, then there exist an extra investor out there who is willing to buy shares at a higher price than the seller paid for. If this happens, the short day EMA will appear to have a higher value than long day EMA since we already assign smaller probability. The value being still bigger, assures that the stock is maintaining positive growth and will be keeping its record to be traded at a higher price. The same reasoning can be applied if the price of the traded asset has decreasing evolution.

Note that if an investor plans to use EMA_n as indicators to analyze market, he will drop the analysis of SMA_n because the process of generating EMA_n has already considered SMA_n.

2. Momentum Hypothesis

The Momentum Hypothesis is dependent on the difference of two SMA_ns or two EMA_ns. As described above, we know that SMA_n or EMA_n captures what happened in the past at a specific time or within a period a time, but it is only one number. In order to understand the momentum of the market, we will look at more than one pair of SMA_n and EMA_n. In the following text, we will discuss EMA_n rather than SMA_n because the calculation of EMA_n already takes SMA_n into account.

2.1 Moving Average Convergence Divergence (MACD)

The definition of Moving Average Convergence Divergence (referred to as MACD in the text) appears in Appel [2]. MACD is defined as the difference between two EMA_n values.

$$Definition\ 3\colon\ MACD = EMA_m - EMA_n$$
$$for\ any\ m < n\ and\ m, n\ \in \mathbb{Z}.$$

The concept of MACD is important in financial investments, since not only does it measure the market momentum for the first time; it also encompasses the evolution of the momentum. This perspective will be frequently used throughout this paper. Based on Hypothesis 1 and Hypothesis 2, we can see that the bigger the MACD is, the more bullish the market can be. However, how bullish can the market be?

2.2 Momentum Hypothesis

The Momentum Hypothesis is essentially an extension of the idea of moving averages. It involves computing an exponential measurement on the MACD and then takes into account the behavior of the time derivatives of these measurements. Appel in [2] only briefly describes the process, so we exhibit here a thorough definition:

$$Definition\ 4\colon We\ defined\ Signal\ Line\ as\ the\ result\ of\ the\ plot\ when\ applying\ an\ 9 - day\ Exponential\ Moving\ Average\ on\ MACD.$$

$$EMA_9\ on\ MACD = MACD * \frac{2}{1+n} + \left(\frac{1}{n}\sum_{i=1}^{n} MACD_i\right) * \left(1 - \frac{2}{1+n}\right)$$
$$for\ any\ n\ \in \mathbb{Z}_+.$$

$$Definition\ 4\colon\ We\ define\ the\ momentum\ of\ a\ price\ action\ to\ be\ the\ first\ derivative\ of\ MACD\ in\ respect\ of\ time, namely$$
$$\frac{\partial MACD}{\partial t} = \frac{\partial (EMA_m - EMA_n)}{\partial t} = \frac{\partial EMA_m}{\partial t} - \frac{\partial EMA_n}{\partial t},$$
$$for\ any\ m < n\ and\ m, n\ \in \mathbb{Z}.$$

We are looking at momentum as the change of MACD, which is generated by describing price action in an exponential way. In other words, we look at momentum as a difference of how fast the exponential description of price action changes.

$$Hypothesis\ 3\colon\ For\ any\ two\ time\ frames\ of\ EMA\ selected\ for\ a\ certain\ stock, the\ higher\ the\ momentum\ the\ higher\ the\ demand\ of\ the\ stock.$$

In order to understand particular price action with extreme “speed” of exhausting the inside offers on a tape, we want to mathematically describe a situation where sellers with the lowest offers can be easily digested by the market disregard the fundamentals of a company. We calculate EMA to describe price and we take the difference of two selected EMAs. We take the first derivative in respect of time of the difference and we obtain momentum of the stock at a particular time. Since the price action is arbitrary, we will have a series of numbers in data describing momentum in respect of time. The hypothesis explains a specific time of price action where the higher the momentum the faster the sellers are exhausted by the market. This means the stock is moving with extreme momentum and likely to be parabolic. Disregard of any fundamental changes, a price action with high momentum can be understood as an irrationally bullish behavior in the secondary market.

Economically, the EMA_9 on MACD is an accurate way to understand how different investors interact when they are at different times, and it takes the length of the time period into consideration. The longer the length between the two times we are comparing, the smaller effect the recent momentum will affect our decision, and the more accurate the result will be expected.

According to Benjamin Graham [3], most investors tend to act somewhere between rationality and irrationality. In other words, you can hardly ever find anybody who is completely rational or completely irrational. The market is like a guy who can be moody today and excited tomorrow, let's call him Mr. Market. He is an interesting guy and the Momentum Hypothesis described in this paper will be trustworthy to be your best tool to unveil what is in Mr. Market's mind. Furthermore, the Momentum Hypothesis will work out better if the general evolution of the market is the one that influences Mr. Market's decision, and not some big player, like the state or some top companies that controls the direction of the market.

3. Introduction to the Regression Analytical Model

The Regression Analytical Model is developed from the Momentum Hypothesis, and gives a way to understand and predict the momentum mentioned in the section above. We will show that RAM can predict an outcome within 0.1% error if the market controls Mr. Market. The concept of market in this paper is mainly referring to all the individual investors and those corporations that are not in top three in their industries. In other words, RAM is aiming to measure an as wide a market as it can measure rather than considering a market that is controlled or moved by only a few big individuals or organizations. In history, RAM could not predict the financial crisis in 2003 and in 2008. However, the period between 2003 and 2008 appears to be accurate by RAM. So given a steady evolution, the RAM gives a fairly accurate description.

Note that what makes the RAM special is that it is not merely a model, it is a way of understanding and predicting momentum and it can be used on any model with sufficient amount of data.

3.1 Regression Analytical Model (RAM)

Instead of giving a definition right away, let us first go through the process of creating the RAM model. It is necessary to use second-hand data as original observations. For example, one can choose daily closing price of SP500 from 2007 to 2012.

Using the observational data, we generate SMA_{12}, EMA_{12}, SMA_{26}, EMA_{26}, MACD, $MACD_9$, Signal, and Delta where $MACD_9$ is the 9-day SMA on MACD and Signal is 9-day EMA on MACD, and Delta is the difference between MACD and Signal Line. In real world, it is not required to have to create a model with the same day period like above. One can choose another number of days depending on the context.

After generating the different indices, we choose the variables we want to predict and shift that list of observation one cell above and import the observations along with the indices calculated and plug it into STATA (a statistical software package). These will be the random variables to which we generate a linear-linear regression function. This is the first group of estimators. We will draw a second group of testable estimator with variables based on experience and compare the R^2 and Mean Square Error between two estimators. We also need to judge the result by omitting variables that do not satisfy 95% confidence interval before we start to compare two groups of testable estimators. After we delete a group, we choose a third group of testable estimator and re-run the regression again; and compare the R^2 and Mean Square Error as well. We repeat this method until we find a group of testable estimator that subjectively satisfies our condition.

After we have our regression model, we are able to generate a specific price based on the second-hand observations. In our example, as mentioned before, the second-hand observation are daily closing price from 2003 to 2013. The result of the calculations done above is an estimate of the closing price of next trading day. As one can imagine, with different choice of second-hand data, one can get predictions at a different time in the future. Choosing, for example, weekly closing prices from 2003 to 2013, then one can calculate the closing price of the following trading week. This whole process is the heart of the RAM model. Furthermore, let us note that it is possible to use other indices as observation group, such as Net Book Value, Earnings per Share, Quick Ratio, Chaikin's Volatility, Slow Stochastic, William's R Curve, and so on.

Our main goal with this procedure it to develop an objective approach subjectively, by analyzing irrational securities rationally. First of all, we need an objective approach because all methods need to make sense in their considerations. However, different periods of time used in the model have different characteristics so one need to use his or her experience to choose which parameters are used to analyze the market.

The securities traded are marketable equities and are related to Graham's irrational Mr. Market. When we are analyzing securities, we are actually analyzing Mr. Market, who is assumed to be irrational. To get to know Mr. Market better, we need to be rational ourselves first. When we observe different interactions among other investors, we should not follow them like the rest of market tends to do, instead, we should keep calm and think about what is happening before we make investment decisions.

3.2 Application with Regression Analytical Model

We will explain the different groups of test based on different models developed from RAM. The first two groups conducted are dependent on the variables from technical indices while the latter two conducted are from variables drawn from Berkshire Hathaway's financial statements from 1996 to now.

3.2.1 Application I

The first test in this paper is a group of observations based on daily closing price S&P 500. We collected the Closing Price of S&P 500 from January 5, 1970 to April 4, 2013 as our observation group (i.e. 10880 observations). As described from previous part, we have calculated SMA_{12}, EMA_{12}, SMA_{26}, EMA_{26}, MACD, $MACD_9$, Signal Line, and Delta (the difference between MACD and Signal Line). We generate a new observation column, namely T1, in our chart by shifting the Closing Price one cell up. The goal of this model is to predict what will happen in the next unit of time, in this case, the next day. We need to understand what is the relation between variables in the past and the corresponding closing price one day in the future. By generating T1, we can accomplish this goal. In the first group of test, we choose three factors as our independent variables: EMA_{12}, EMA_{26}, and Delta. Thus, we have the following table:

T1	Coefficient	STD Error	T-value	95% Confidence Interval	
EMA-12	1.2975	.0110	117.98	1.2759	1.3190
EMA-26	-0.2972	0.0110	-26.99	0.3188	0.2756
Delta	1.4559	0.0332	46.67	1.3948	1.5170
Constant	0.3138	0.1911	1.64	0.0608	0.6884

From the coefficient of the variables generated from STATA, we can generate the following equation:

$$EstimatedPrice = 1.2975 * EMA_{12} + (-0.2972) * EMA_{26} + 1.4559 * Delta + .3138$$

We can generalize the previous equation, and we have the following:

$Momentum\ Regression\ Analytical\ Model$ 1:
$$Estimated\ Price = \alpha * EMA_{12} + \beta * EMA_{26} + \gamma * Delta + \varepsilon \ \ while\ \alpha, \beta, \gamma, and\ \varepsilon\ are\ constants.$$

After we have an equation to help us understand the relationship between Price and EMA-12, EMA-26 and Delta, we can plug into the numbers we know from today's market moving. Thus, we have the following estimated predicted price calculated. Predicted Closing Price (for April 5, 2013, i.e. the following trading day after April 4, 2013) = 1.2975*1558.822-0.2972*1550.502+1.4559*(-5.1776)+0.3138 = $1554.5381per share. Furthermore, we can check the closing price of April 5, 2013, which is $1553.28 per share, with an error at 0.081% (computed by 1554.5381/1553.28-1 which means the predicted price is 0.081% away from the actual price, and it is the same throughout this paper). This looks like an accurate result.

However, with the knowledge of econometrics, we can make the result even more accurate! We can conduct a t-test for the coefficient of the Constant in the regression model, we set up our null hypothesis, "the coefficient of Constant is zero", and the alternative hypothesis will be the opposite of "the coefficient of Constant is not zero". We check the t-value, which is 1.64, which is less than 1.96 at 95% confidence interval. Thus, we cannot reject null hypothesis. In this case, we can drop the constant in our equation. It turns out to be as follows: 1.2975*1558.822 - 0.2972*1550.502 + 1.4559*(-5.1776) = $1554.2243 per share. This will give us an error at 0.061%.

3.2.2 Application II

Although the previous results look promising, the RAM model we just created cannot be used all the time. This is because, as time passes, the investors will experience different financial behavior and their psychology will make them turn opposite the previous evolution. That is why we are introducing our second model based on RAM.

For example, if we were on April 10, 2013, if we kept using the previous model, we would generate a very inaccurate result. Let us run the regression model with EMA-12, EMA-26, and Delta as independent variables just as above. We will have the following:

T1	Coefficient	STD Error	T-value	95% Confidence Interval	
EMA-12	1.2978	0.0110	118.01	1.2762	1.3193
EMA-26	-0.2975	0.0110	-27.02	0.3191	0.2759
Delta	1.4545	0.0312	46.64	1.3934	1.5156
Constant	0.3078	0.1911	1.61	0.0669	0.6824

Then from the equation generated by RAM, we will have 1.2978*1567.63-0.2975*1559.631+1.4545*(-1.6713)+0.03078 = $1568.0797 per share. As we expected, the result is a lot different from the actual result, because the actual result is $1593.37 per share. Not only did the estimator gave a wrong direction of future price, it also produced a huge estimated error, an error that cannot even be covered from the model within 95% confidence interval. How do we solve this problem?

We introduce our second model developed from RAM. Instead of predicting T1, we switch our minds to predict SMA-12 with one cell shifted above. That is to say, we want to set SMA-12 as our dependent variable, and we want to predict the next trading day's SMA-12. We can then calculate backwards to get tomorrow's closing price. In order to do this, we cannot keep using the same independent variables either. But the same logic applies in all RAM developed models. We are trying to predict SMA-12, which is why we need to use simple averages as our factors, so we want to treat SMA-26 and MACD-9 (which is 9-day SMA on MACD) as our independent variables. Here is the regression model generated by STATA:

T1SMA12	Coefficient	STD Error	T-value	95% Confidence Interval	
SMA-26	0.9998	0.0001	8018.57	0.9996	1.0000
MACD-9	1.0615	0.0055	193.83	1.0508	1.0723
Constant	0.2137	0.0959	2.23	0.026	0.4017

After we have our regression model developed from RAM, we can calculate the future closing SMA-12, which is 0.9998*1557.383+1.0615*9.9877+0.2137 = $1567.8872 per share. Then we can calculate backwards to get the predicted closing price. We compute 1567.8872*12 – SUM (B10909 to B10919) = $1600.0564 per share. If we compare this predicted price with the actual losing price $1593.37, then we will have an error of 0.42%, which is less accurate from previous model but at least it gives us the right direction of the market moving trend. This also proves what as we have discussed before, that the less rational the market is, the less accurate RAM can be.

The second model developed from RAM gave us the following generalized equation.

$$Equation\ 2: Estimated\ SMA_n = \alpha * SMA_m + \beta * MACD_9 + \varepsilon\ \ while\ \alpha, \beta, \varepsilon\ are\ constants, and\ for\ any\ n > m\ chosen\ subjectively.$$

3.2.3 Application III

One can get a sense why RAM is so special: it enables us to connect various situations and build thousands of new individual models stemmed from the original.

To extend the idea, the previous two applications can be helpful on analyzing individual stocks. However, one needs to make sure the situation is fully analyzed before applying the model. RAM developed models by using technical indices as factors may not be suitable for steadily growing companies; but there is no need worry. We will introduce a third model that will help us solve this aspect.

The third model will include basic understanding of financial accounting. However, instead of using the logic provided by RAM on closing prices, we apply it on financial factors. From *Security Analysis*, the understanding of intrinsic value is introduced in connection to the concept of Net Book Value. Analyzed by Benjamin Graham, the definition of Net Book Value is as follows:

$$Definition\ 4: Net\ Book\ Value = Common\ Stock + Surplus - Intangibles$$

$$Definition\ 5: Net\ Book\ Value\ Per\ Share = \frac{Common\ Stock + Surplus - Intangibles}{Share\ of\ Common\ Stock\ Outstanding}$$

The definition of Net Book Value is not just an equation to calculate the intrinsic value of a company; it is rather a way of thinking about how much a company is worth. Benjamin introduced in [3] the idea of value investing by choosing lower price-to-book ratio companies, i.e. investing in value growth.

With this idea in mind, we are going to go through the process of how this third model is developed, and why it is important. The example we used depends on Berkshire Hathaway's annual and seasonal report started from 1996 up to 2011. Suppose we want to predict the price at the beginning of 2012, so we are putting ourselves back in the time. We will create models based on information that collected before and excluding 2012, and we will make a prediction on closing prices for the year 2012. We start by thinking about a company's book value. What factors should be affecting a company's book value? The first thing comes in mind is Cash. For any individual organization, the money in the pocket right now that is free for liquidation and for any utilization or re-investment is the core value and what the organization has right now. It is apparent to anybody that the larger the Cash is; the better the company stands financially. Furthermore, we want to consider Net Book Value as well. In this example, we calculated Net Book Value by computing (Retained Earnings + Capital Excess – Goodwill), but this equation may change when we are facing different companies. Thus, we have the regression model as below:

PSMAUP	Coefficient	STD Error	T-value	95% Confidence Interval	
CSMA3	0.4039	0.1061	3.81	0.1916	0.6163
NSMA3	1.2339	0.1195	10.33	0.9947	1.4731
Constant	41948.68	2420.994	17.33	37102.54	46794.83

Note that PSMAUP means Price with SMA-3 updated and shift one cell up, CSMA3 means Cash with SMA-3 updated, and NSMA3 means Net Book Value updated with SMA-3. Thus, we have an estimated regression function for Price with SMA-3 updated. We can then calculate the future price by using 2011 Cash with SMA-3 updated and Net Book Value with SMA-3 updated. We compute 0.4039*39988.67+1.2339*56109.95+41948.68 = $127,334.17 per share. From the model, this means that 2012 Closing Price on A stock of Berkshire Hathaway is supposed to be $127343.17 per share. We check that the actual price is $134,060 per share. Thus, we have an estimator with an error at 5.28%. We are not done yet. We still have a chance to increase the accuracy of this result.

However, we have another problem here. We treat the previous example a little bit differently than the first and second model developed from RAM. The reason is that we are now selecting independent variables from financial report, and the result will not only be a prediction, but rather how much the company is supposed to be valued by the market. We need to pay attention to the time difference.

The data we used in this example has observations listed in time order. For each year, we have listed Q1 to Q3 and 10-K, but there is a time delay in the report. The company will not post Q1 specifically in the end of each quarter; there may be a month of time delay. In our example, Berkshire Hathaway posted 10-K the next following year in March, thus we have a three-month delay. In other words, based on regression theory in study of econometrics, the $127334.17 per share is actually the estimated price of June or July in the year 2012 for Berkshire Hathaway. If Berkshire Hathaway publishes the first seasonal report of 2012 at the end of June, the actual price $127445 per share. Thus, we will have a result that is 0.087% away from the real number. In real financial market, a prediction three month in the future with an estimated price that is 0.087% away from actual price is very accurate!

Thus, it is apparent that we have moved away from Momentum Hypothesis and technical indicators already. This is why Momentum Regression Analytical Model is changing each time into powerful tools. By choosing different parameters, we can create all sorts of models. The last application we introduce will include the understanding of price-to-book ratio mentioned above. We look at Berkshire Hathaway historical financial positions from 2003 to 2012 annual reports. We can draw the following table. As shown, annual price is presented with corresponding Net Book Value and Price-to-Book Ratio. Following the same logic, we shift the price one cell up in column "Price+1". Next we draw regression by choosing Net Book Value and Price-to-Book as parameters. This makes sense because Net Book Value does contribute to Price per share. The result shows a positive correlation, and this is because Berkshire's Net Book Value has been contributed and it is also dependent to retained earnings. The more earnings a corporation can retain, the more re-investment the corporation will be doing, and thus more likely to create value. Price-to-Book Ratio is another indicator. Graham (1934) explained that Price-to-Book Ratio tells investors how "expensive" a share is. Comparing this financial indicator to peers or to market, we can find more intriguing results. We thus introduce a two-factor model here.

$Two - factor\ Hypothesis_1$:

$$Price = \alpha * Net\ Book\ Value + \beta * \frac{P}{B} Ratio + \epsilon,$$

$for\ some\ \epsilon \in \mathbb{R}\ and\ \frac{P}{B}\ refers\ to\ Price\ per\ Book\ Value.$

This model is unique in a way that it takes Net Book Value and P/B Ratio into account at the same time. What it explains is that we can understand a security or a particular portfolio's performance in the market by looking at Net Book Value and P/B Ratio.

With some understanding of the market or a simple comparison, one can even draw correlation between market value and value of a particular security. Thus, we can take derivative on each side and have the following hypothesis.

$Two - factor\ Hypothesis_2$:

$$\frac{\partial Price}{\partial t} = \alpha \frac{\partial(Net\ Book\ Value)}{\partial t} + \beta \frac{\partial\left(\frac{P}{B}\right)}{\partial t} + \epsilon,$$

$for\ some\ \epsilon \in \mathbb{R}\ and\ \frac{P}{B}\ refers\ to\ Price\ per\ Book\ Value.$

This model is a developed model from the first hypothesis such that it looks at the change of each parameter corresponding to the change of price, i.e. the return of a security. This is another powerful tool because it has the ability to take an imagined market opinion into account.

RAM model can be applied into different generations by selecting different parameters depending on the needs of each financial analyst subjectively according to different situation.

Date	Price	Price+1	Return	Net Book Value	Price to Book
2003-12	82,800	87,900	-	58,032	1.4268
2004-12	87,900	88,620	0.0616	65,457	1.3429
2005-12	88,620	109,990	0.0082	74,116	1.1957
2006-12	109,990	134,000	0.2411	85,434	1.2874
2007-12	134,000	93,400	0.2183	99,105	1.3521
2008-12	93,400	100,300	(0.3030)	105,305	0.8869
2009-12	100,300	120,450	0.0739	113,301	0.8853
2010-12	120,450	114,755	0.2009	136,727	0.8810
2011-12	114,755	133,000	(0.0473)	147,255	0.7793
2012-12	133,000	-	0.1590	161,502	0.8235
2013-12		-		174,840	0.9766

Price	Coef.	Std. Err.	T	P>|t|	[95% Conf. Interval -]	[95% Conf. Interval +]
Net Book Value	0.9855822	0.124862	7.89	0	0.6903305	1.280834
Price-to-Book	92767.46	17186.33	5.4	0.001	52128.24	133406.7
Const.	-97431.69	30862.1	-3.16	0.016	-170409	-24454.42

4. Conclusions

Investors can be moody or excited all of a sudden at any time, and their emotional state can influence their move on the market. Although economists assume that each human being in the market is rational, it is essential to understand that the main assumption in this paper is that market is irrational. And our model builds an objective approach to subjective phenomena and allows us to analyze irrational securities in a rational way.

Once an investor collects enough unbiased information for one corporation, he or she can conduct or develop a model from RAM.

RAM is not just accurate, but it gives a completely new perspective. It shows us a way one can build equations and models that one can use in the future to predict results that are fairly accurate.

In particular, we built different models from RAM based on traditional investment. We collected independent variables from books and tried to find patterns between price and factors recorded in book, such as Total Assets, Net Earnings, and Retained Earnings etc.

6.4 Future Perspective

As a final remark, the methods described in this paper could assist the reader in their investment decision. When an investor has enough knowledge, he can develop his own model based on the Regression Analytical Model.

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