Art of Money Management

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Abstract

This document preserves the decision-making process of author himself when making investment and trading decisions. The essence of this document rests in the implementation of machine power as extensions of human will.

This document is dedicated to my parents and all my friends.

Preface

Inspired by the most influential martial artist, Bruce Lee, I have never been a big fan of "style". Style means limiting oneself to a series of repeated formats and events, which I found impractical in real life. You want to choose your own upper bound, and please be aware: infinity is an option. A good fighter should be familiar with a whole range of styles so that he can face and survive any fights with any enemy. I believe the art of money management shares the exact same principle.

I found money management a combination of macroeconomics, microeconomics, international trade, political science, game theory, cultural integration, asset management, anomaly identification, pattern recognition, trading, sales and trading, fundamentals, technicals, and so on. A money manager should almost never lean toward or prefer one or the other unless he has solid evidence that his information is earlier and more accurate than that of the market. In other words, one needs to be able to integrate every available knowledge to be able to understand the art of money management. In this document, I will explain how they (i.e. macroeconomics, fundamentals, asset pricing, statistics theory, and game theory) all play together to form a big decision making map.

Enjoy!

Yiqiao Yin

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1 §Introduction§

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Given a certain amount of wealth, how does an individual manage the wealth in an efficient manner? To answer this question, *The Art of Money Management* presents its readers the following philosophy: *The art of money management rests in creating, integrating, specializing an objective approach subjectively on analysis of irrational market rationally.*

This document takes the following order.

Section Two §*2 Macroeconomics* uses a trade dynamic model to identity inflow and outflow of block trades. I take the benchmark model from Afonso and Lagos (2015) [1].

Section Three §*3 Fixed Income and Derivatives* presents definitions of common financial products.

Section Four §*4 Fundamentals* provides a list of financials and accounting terms to "value" a stock. Section Three and Four provide background for Section Five.

Section Five §*5 Asset Pricing* looks at the stock data from an empirical view. In this section, I propose a model studying the greed of the underlying market. Instead of claiming a top in a stock or in one market, I defined "greed" to be a relative term. An investor can pull all the data together and use this model to measure whether one security (stocks, portfolio, etc.) has more investors chasing prices than the other.

Section Six §*6 Price as a Martingale* provides readers my understanding of random walk.

Section Seven §*7 Trading*, discusses some theories (accompanied with data) to show that how to identify actionable prices from prior distribution. It is essential that we understand not only what to buy, but also when to buy and how to buy? This section introduces the essence of this paper: *Alpha Protocol*, *House Party Protocol*, and *The Winter Contingency*. I propose a method to look at equity prices. Under this method, statistically unlikely prices can be identified from prior distribution. New data that rests within the range of prior distribution calls for *Alpha Protocol* if certain thresholds are hit. New data rests outside the prior distribution will be collected to generate posterior distribution. In this case, data that rests above prior distribution calls for *House Party Protocol*. Data that rests below prior distribution calls for *The Winter Contingency*.

Section Eight, §*8 Game Theory*, provides us the understanding of playing games with market movers.

As a summary, this document zooms in from macro- to micro- point of view to study this market, presenting a systematic approach to help readers making more sound money management decisions.

2 §Macroeconomics§

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Financial institutions in US keep reserve balances at Federal Reserve Banks to meet requirements, earn interest, or clear financial transactions. This market allows institutions with excess reserve balances to lend reserves to institutions with reserve deficiencies. The fed funds market is primarily a mechanism that reallocates reserves among banks. It is a crucial market from the standpoint of the economics of payments and the branch of banking theory that studies the role of interbank markets in helping banks manage reserves and offset liquidity or payment shocks. The fed funds market is also the setting where the interest rate on the shortest maturity, most liquid instrument in the term structure is determined. This makes it an important market from the standpoint of finance.

2.1 Trade Dynamics

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In the field of studying fed funds market, Poole (1968) [40], Ho and Saunders (1985) [26], and Coleman, Gilles, and Labadie (1996) [10] contributed in theoretical perspective whereas Hamlton (1996) and Hamlton and Jorda (2002) [23] contributed in empirical perspective. Ashcraft and Duffie (2007) [3] highlighted the over-the-counter nature of the fed funds market. Bech and Klee (2011) [6], Ennis and Weinberg (2009) [16], and Furfine (2003) [19] used this nature to try to explain certain aspects of interbank markets such as apparent limits to arbitrage, stigma, and banks' decisions to borrow from standing facilities. Afonso and Lagos (2015) [1] contributed a model to the intraday allocation of reserves and pricing of overnight loans using a dynamic equilibrium search-theoretic framework such that it can capture the salient features of the decentralized interbank market in which these loans are traded. Their work arises from emerging literature that studies search and bargaining frictions in financial markets.

The population of banks, represented by a point in the interval [0,1]. During trading session, the reserve balances set in continuous time that starts at time 0 and ends at time T. Let τ denote the time remaining until the end of trading session, so $\tau = T - t$ if the current time is $t \in [0, T]$. The reserve balance at time $T - \tau$ is denoted by $k(\tau) \in \mathbb{K}$, with $\mathbb{K} = \{0, 1, ..., K\}$, where $K \in \mathbb{Z}$ and $1 \leq K$. The measure of banks with balance k at time $t - \tau$ is denoted $n_k(\tau)$. A bank starts the trading session with some balance $k(T) \in \mathbb{K}$. The initial distribution of balances, $\{n_k(T)\}_{k \in \mathbb{K}}$, is given. Let $u_k \in \mathbb{R}$ denote the flow payoff to a bank from holding k balances during the trading session, and let $U_k \in \mathbb{R}$ be the payoff from holding k balances at the end of the trading session. All banks discount payoffs at rate r.

Afonso and Lagos (2015) assume that loan sizes are elements of the set $\bar{\mathbb{K}} = \mathbb{K} \cup \{-K, ..., -1\}$ and that every loan gets repaid at time $T + \triangle$ in the following trading day, where $\triangle \in \mathbb{R}_+$. Let $x \in \mathbb{R}$ denote the net credit position that has resulted from some history of trades. Afonso and Lagos (2015) also assume that the payoff to a bank with a net credit position x that makes a new loan at time $T - \tau$ with repayment R at time $T + \triangle$ is equal to the post-transaction discounted net credit position, $e^{-r(\tau+\triangle)}(x + R)$.

Afonso and Lagos (2015) [1] abstract brokers from the baseline model since nonbrokered transactions represent the majority of the volume. The search entails two layers of uncertainty in this environment. First, the time it takes a bank to contact a counterparty is an exponentially distributed random variable with mean $1/\alpha$. Second, conditional on having contacted a counterparty at time $T - \tau$, the reserve balance k of the counterparty is a random variable with probability distribution $\{n_k(\tau)\}_{k \in \mathbb{K}}$.

Let the function $V_k(\tau) : \mathbb{K} \times [0, T] \to R$ denote the maximum attainable payoff that a bank can obtain from $k \in \mathbb{K}$ units of reserve balances when the time until the end of the trading session is $\tau \in [0, T]$. During the trading hours, two banks meet and bargain over size of the loan and repayment. Consider a bank with k balances that contacts a bank with k' balances. For any pair of pre-trade reserve balances of two banks, $k, k' \in \mathbb{K}$, the set $\Pi(k, k') = \{(q, q') \in \mathbb{K} \times \mathbb{K} : q + q' = k + k'\}$ contains all feasible pairs of post-trade balances that could result from the bilateral negotiation. For every pair of banks that hold pre-trade balances $(k, k') \in \mathbb{K} \times \mathbb{K}$, the set $\Pi(k, k')$ induces the set of all feasible loan sizes, $\Gamma(k, k') = \{b \in \bar{\mathbb{K}} : (k - b, k' + b) \in \Pi(k, k')\}$. Notice that $\Pi(k, k') = \Pi(k', k)$ and $\Gamma(k, k') = -\Gamma(k', k)$ for all $k, k' \in \mathbb{K}$. Then we have $b_{kk'}(\tau)$ to be the amount of reserves that the bank with k lends to the bank with balance k', and $R_{k'k}(\tau)$ to be the amount of balances that the latter commits to repay the former at time $T + \triangle$. Thus the pair $(b_{kk'}(\tau), R_{k'k}(\tau))$ denotes the bilateral terms of trade between a bank with balance k and a bank with balance k' when the remaining time until the end of the trading session is τ. These terms of trade will be the outcome corresponding to the symmetric Nash solution to the bilateral bargaining problem. Then, for any $k, k' \in \mathbb{K}$ and any $\tau \in [0, T]$,

$(b_{kk'}(\tau), R_{k'k}(\tau))$ is the solution to

$$\max_{b\in\Gamma(k,k'),R\in\mathbb{R}}[V_{k-b}(\tau)+e^{-r(\tau+\triangle)}R-V_k(\tau)]\times[V_{k'+b}(\tau)-e^{-r(\tau+\triangle)}R-V_{k'}(\tau)].$$

Thus, for any $k\in\mathbb{K}$ and any $\tau\in[0,T]$,

$$V_k(\tau)=\mathbb{E}\Big\{\int_0^{min(\tau_\alpha,\tau)}e^{-rz}u_k dz+\mathbb{I}_{\{\tau_\alpha>\tau\}}e^{-r\tau}U_k+$$

$$\mathbb{I}_{\{\tau_\alpha\le\tau\}}e^{-r\tau_\alpha}\sum_{k'\in K}n_{k'}(\tau-\tau_\alpha)\times[V_{k-b_{kk'}(\tau-\tau_\alpha)}(\tau-\tau_\alpha)+e^{-r(\tau+\triangle-\tau_\alpha)}R_{k'k}(\tau-\tau_\alpha)]\Big\} \quad (1)$$

where

$$b_{kk'}(\tau)\in arg\max_{b\in\Gamma(k,k')}[V_{k'+b}(\tau)+V_{k-b}(\tau)-V_{k'}(\tau)-V_k(\tau)], \quad (2)$$

$$e^{-r(\tau+\triangle)}R_{k'k}(\tau)=\frac{1}{2}[V_{k'+b_{kk'}(\tau)}(\tau)-V_{k'}(\tau)]+\frac{1}{2}[V_k(\tau)-V_{k-b_{kk'}(\tau)}(\tau)]. \quad (3)$$

The expectation operator, $\mathbb{E}$, in (1) is with respect to the exponentially distributed random time until the next trading opportunity, τ_α, and $\mathbb{I}_{\tau_\alpha\le\tau}$ is an indicator function that equals 1 if $\tau_\alpha\le\tau$ and 0 otherwise. The first term contains the flow payoff to the bank from holding balance k until the next trade opportunity or the end of the session, whichever arrives first. The second term says that in the event that the bank gets no trading opportunity before time T. The third term contains the expected discounted payoff if the event that the bank gets a trade opportunity with another bank before time T, that is, at time $T-(\tau-\tau_\alpha)$. In this event, the counterparty is a random draw from the distribution of balances at time $T-(\tau-\tau_\alpha)$, namely, $\{n_{k'}(\tau-\tau_\alpha)\}_{k'\in\mathbb{K}}$, and the expression inside the square bracket is the post-trade continuation payoff of the bank we are considering. Hereafter, Afonso and Lagos (2015) use $\mathbf{V}\equiv[\mathbf{V}(\tau)]_{\tau\in[0,T]}$, with $\mathbf{V}(\tau)\equiv\{V_k(\tau)\}_{k\in\mathbb{K}}$ to denote the value function. According to the bargaining solution (2) and (3), the loan size maximizes the joint gain from trade, and the repayment implements a division of this gain between the borrower and the lender that gives each a fraction equal to their bargaining power (that is, one half). For example, if a bank with $i\in\mathbb{K}$ balances and a bank with $j\in\mathbb{K}$ balances meet at time $T-\tau$, they will negotiate a loan of size $b_{ij}(\tau)=i-k=s-j$, where $(k,s)\in arg\ max_{(k',s')\in\Pi(i,j)}S_{ij}^{k's'}(\tau)\equiv V_{k'}(\tau)+V_{s'}(\tau)-V_i(\tau)-V_j(\tau)$. The implied joint gain from trade corresponding to this transaction is $S_{ij}^{ks}(\tau)$.

Consider a bank with i balances that contracts a bank with j balances when the time until the end of trading session is τ. Let $\phi_{ij}^{ks}(\tau)$ be the probability that the former and the latter hold k and s balances after the meeting, respectively, that is, $\phi_{ij}^{ks}(\tau)\in[0,1]$, with $\sum_{k\in\mathbb{K}}\sum_{s\in\mathbb{K}}\phi_{ij}^{ks}(\tau)=1$. Feasibility requires that $\phi_{ij}^{ks}(\tau)=0$ if $(k,s)\notin\Pi(i,j)$. Given any feasible path for the distribution of trading probabilities, $\phi(\tau)=\{\phi_{ij}^{ks}(\tau)\}_{i,j,k,s,\in\mathbb{K}}$, the distribution of balances at time $T-\tau$, that is, $n(\tau)=\{n_k(\tau)\}_{n\in\mathbb{K}}$, evolves according to

$$\dot{n}_k(\tau)=f[\mathbf{n}(\tau),\phi(\tau)]\ \forall k\in\mathbb{K}, \quad (4)$$

where

$$f[\mathbf{n}(\tau),\phi(\tau)]\equiv\alpha n_k(\tau)\sum_{i\in\mathbb{K}}\sum_{j\in\mathbb{K}}\sum_{s\in\mathbb{K}}n_i(\tau)\phi_{ki}^{sj}(\tau)-\alpha\sum_{i\in\mathbb{K}}\sum_{j\in\mathbb{K}}\sum_{s\in\mathbb{K}}n_i(\tau)n_j(\tau)\phi_{ij}^{ks}(\tau). \quad (5)$$

The first term on the right side of (5) contains the total flow of banks that leave state$k$ between time $t=T-\tau$ and time $t'=T-(\tau-\epsilon)$ for a small $\epsilon>0$. The second term contains the total flow of banks into state k over the same interval of time.

The following proposition provides a sharper representation of the value function and the distribution of trading probabilities characterized in (1), (2), and (3).

Proposition 1: *The value function* $\mathbf{V}$ *satisfies (1), with (2) and (3), if and only if it satisfies*

$$rV_i(\tau)+\dot{V}_i(\tau) = u_i+\frac{\alpha}{2}\sum_{j\in\mathbb{K}}\sum_{k\in\mathbb{K}}\sum_{s\in\mathbb{K}} n_j(\tau)\phi_{ij}^{ks}(\tau)\times[V_k(\tau)+V_s(\tau)-V_i(\tau)-V_j(\tau)]\ \forall(i,\tau)\in\mathbb{K}\times[0,T] \tag{6}$$

with

$$V_i(0) = U_i\ \forall i\in\mathbb{K} \tag{7}$$

and

$$\phi_{ij}^{ks}(\tau)\begin{cases}\le 0\ \ if\ \ (k,s)\in\Omega_{ij}[\mathbf{V}(\tau)]\\ =0\ \ if\ \ (k,s)\notin\Omega_{ij}[\mathbf{V}(\tau)]\end{cases}, \forall i,j,k,s\in\mathbb{K}, with \sum_{k\in\mathbb{K}}\sum_{s\in\mathbb{K}}\phi_{ij}^{ks}(\tau)=1, \tag{8}$$

where

$$\Omega_{ij}[\mathbf{V}(\tau)] \equiv arg \max_{(k',s')\in\Pi(i,j)}[V_{k'}(\tau)+V_{s'}(\tau)-V_i(\tau)-V_j(\tau)] \tag{9}$$

The set $\Omega_{ij}[\mathbf{V}(\tau)]$ defined in (9) contains all the feasible pairs of post-trade balances that maximize the joint gain from trade between a bank with i balances and a bank with j balances that is implied by the value function $\mathbf{V}(\tau)$ at time $T-\tau$. For any pair of banks with balances i and j, $\phi_{ij}^{ks}(\tau)$ defined in (8) is probability distribution over the feasible pairs of post-trade balances that maximize the bilateral gain from trade. Thus, (8) and (9) together describe the pairs of post-trade balances (or equivalently, loan sizes) that may result from the bilateral bargaining. The Bellman equation described by (6) and (7) has a natural interpretation. The flow value of a bank that holds balance i at time $T-\tau$, that is, $rV_i(\tau)$, consists of the flow return from holding balance i, that is, u_i, minus the flow capital loss associated with the reduction in the remaining time until the end of the trading session, that is, $\dot{V}_i(\tau)$, plus the rate at which the bank meets counterparties, α, times the expected gain from trade to the bank, that is, $\sum_{j\in\mathbb{K}}\sum_{k\in\mathbb{K}}\sum_{s\in\mathbb{K}} n_j(\tau)\phi_{ij}^{js}(\tau)\frac{1}{2}S_{ij}^{ks}(\tau)$.

Definition 1: An equilibrium is a path for the distribution of reserve balances, $\mathbf{n}(\tau)$, a value function, $\mathbf{V}$, and a path for the distribution of trading probabilities, $\phi(\tau)$, such that, (a) given the value function and the distribution of trading probabilities, the distribution of balances evolves according to (4); and (b) given the path for the distribution of balances, the value function satisfies (6) and (7), and the distribution of trading probabilities satisfies (8).

In quantitative implementations of the theory, one can try to compute equilibrium algorithmically, as follows. Guess a path of trading probabilities and use it to solve the system of differential equations (4) with initial condition $\{n_k(T)\}_{k\in\mathbb{K}}$ to obtain a path for the distribution of reserve balances. Then substitute the trading probabilities and distribution of reserves implied by the guess, and solve the system of differential equations (6) and (7) for the implied value function. Then use this value function and (8) to obtain a new guess for the path of trading probabilities and continue iterating until the value function has converged. Instead of following this route, Afonso and Lagos (2015) make a curvature assumption on the vectors$\{u_k\}_{k\in\mathbb{K}}$ and $\{U_k\}_{k\in\mathbb{K}}$ that will allow us to provide an analytical characterization of equilibrium.

Assumption A: $\forall i,j\in\mathbb{K}$, *the payoff functions satisfy*

$$(DMC)\ u_{\lceil (i+j)/2\rceil}+u_{\lfloor (i+j)/2\rfloor}\le u_i+u_j,$$

$$(DMSC)\ U_{\lceil (i+j)/2\rceil}+U_{\lfloor (i+j)/2\rfloor}\le U_i+U_j,$$

$$\text{“}>\text{''}\ \ unless\ i=\left\lceil\frac{i+j}{2}\right\rceil\ and\ j=\left\lfloor\frac{i+j}{2}\right\rfloor,$$

$$where\ \lfloor x\rfloor\equiv\max\{k\in\mathbb{Z}:k\le x\}\ and\ \lceil x\rceil\equiv min\{k\in\mathbb{Z}:x\le k\}\forall x\in\mathbb{R}.$$

Conditions (DMC) and (DMSC) require that $\{u_k\}_{k\in\mathbb{K}}$ and $\{U_k\}_{k\in\mathbb{K}}$ satisfy the *discrete midpoint concavity property* and the *discrete midpoint strict concavity property*, respectively. These conditions are the natural discrete approximations to the notions of *midpoint concavity* and *midpoint strict concavity* of ordinary functions defined on convex sets. Assumption A is reasonable in the context of the fed funds market because central banks typically do not offer payment schemes that are convex in reserve balances. The following result provides a full characterization of equilibrium under this assumption.

PROPOSITION 2: *Let the payoff functions satisfy Assumption A. Then:*

(i) *An equilibrium exists, and the equilibrium paths for the maximum attainable payoffs,* $\mathbf{V}(\tau)$*, and the distribution of reserve balances,* $\mathbf{n}(\tau)$*, are uniquely determined.*

(ii) *The equilibrium path for the distribution of trading probabilities,* $\phi(\tau) = \{\phi_{ij}^{ks}(\tau\}_{i,j,k,s,\in\mathbb{K}}$*, is given by*

$$\phi_{ij}^{ks}(\tau)\begin{cases}\le 0 \ if \ (k,s)\in\Omega_{ij}^* \\ \\ =0 \ if \ (k,s)\notin\Omega_{ij}^*\end{cases}, \forall i,j,k,s\in\mathbb{K} \ and \ \forall\tau\in[0,T], \ with \sum_{(k,s)\in\Omega_{ij}^*}(\tau)=1, and \tag{10}$$

$$\Omega_{ij}^* = \begin{cases}\left\{\left(\frac{i+j}{2}\,\frac{i+j}{2}\right)\right\} \ if \ i+j \ is \ even, \\ \left\{\left(\left\lfloor\frac{i+j}{2}\right\rfloor,\left\lceil\frac{i+j}{2}\right\rceil\right),\left(\left\lceil\frac{i+j}{2}\right\rceil,\left\lfloor\frac{i+j}{2}\right\rfloor\right)\right\} \ if \ i+j \ is \ odd.\end{cases} \tag{11}$$

3 §Fixed Income and Derivatives§

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3.1 Futures

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Traders can negotiate terms of interest rate and size of the transactions in the market. The action results in a contract stating the buyer of the contract has the right to buy a certain number of shares of something (can be commodities or market index etc.) at a pre-determined price. This is a futures contract. Between present value (PV) and future value (FV), there are the following relations:

$$Annually \ rate \ r : PV = \frac{FV}{(1+r)^n}$$

$$Quarterly \ rate \ r : PV = \frac{FV}{(1+\frac{r}{4})^{4n}}$$

$$Continuously \ rate \ r : PV = \frac{FV}{e^{r(t+\triangle t)}}$$

Futures prices are usually quoted online for free. Most common source is CME Group.

3.2 Forward

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Imagine the following situation, we are at time (t=0),

	Derivative	t=0	t=1	t=2
Payoff	Future in 1-y	PV = 1	FV = 1.05	
Payoff	Future in 2-y	PV = 1	0	FV = 1.06^2
Payoff	Forward from year 1 to year 2	0	1+?	

This is to say that we have three contracts. The first two are futures and the last one is a forward. Each of the futures contract are zero-coupon bond (ZCB). By its name, they pay no coupons. That means the only payment is the payment at its maturity. This is only for simplicity reasons so that people in the industry can communicate better. We say that we have ZCB paying in a year and another ZCB paying in two years.

A forward contract is essentially a futures contract but starting sometime in the future and extend into the future. We have present day at t=0 and two futures' time spot with t=1 and t=2. A forward contract is priced so that people cannot make arbitrage between ZCB of 1-y and ZCB of 2-y. Tony can buy a future of ZCB of 1-y and buy another forward from 1 to 2. Then Mary can buy directly a ZCB of 2-y. These two payments should be exactly the same.

What if they are not the same? For example, if Tony's value in hand is smaller than Mary's value in hand, then Eve can come into this market to buy the futures of ZCB of 1-y and forward contract from Tony and the same time he can sell short Mary's ZCB 2-y. The transactions will cancel off the payment but Eve is left with the difference.

3.3 Contango and Backwardation

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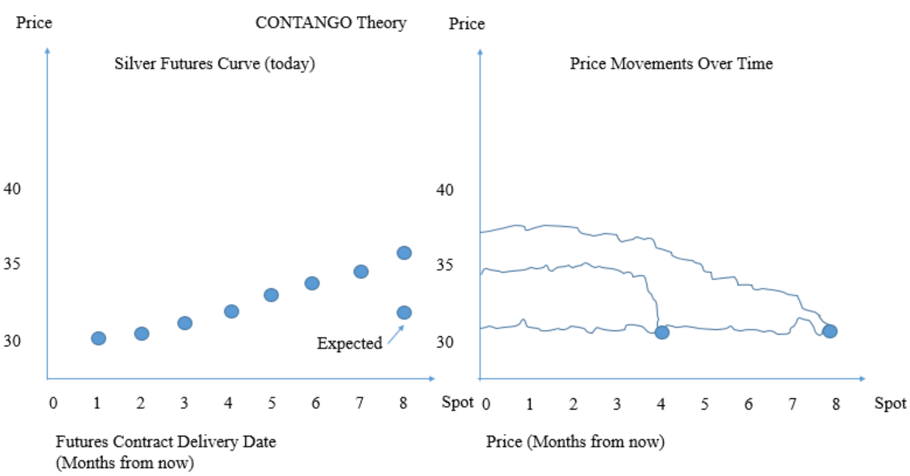
Contango is a situation describing the fact that the currently traded price of a futures contract is higher than expected value of the contracts. This means that sometime in the future the price in the market will converge to the spot price (price of the ZCB) when the time is closer to the maturity date.

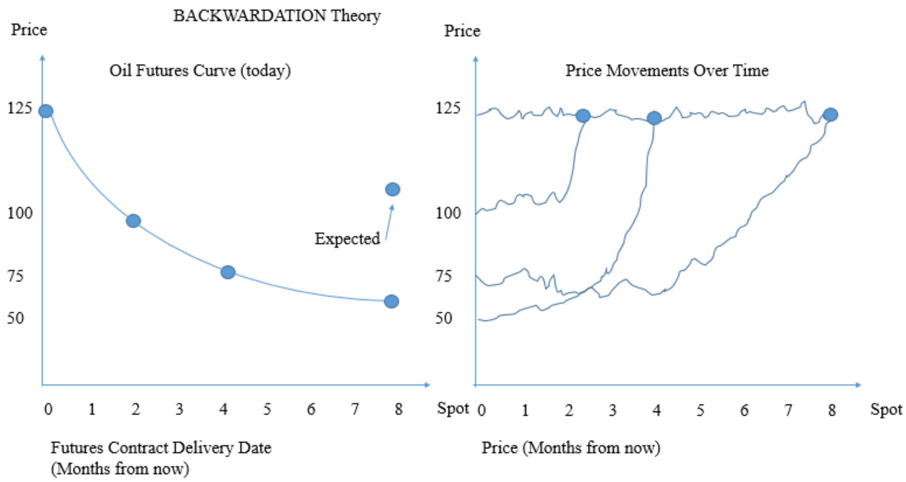
Figure 1: This is graphic illustration for Contango.



Backwardation, the opposite as Contango, is describing a situation where the currently traded price of a futures contract is lower than expected value of the contracts. This refers to a fact in the future that the price in the market will go back up to the spot price (price of ZCB) when time is closer to the maturity date.

I personally do not recommend people to trade off this strategy. It is usually very costly and hard to exploit this opportunity. However, there are indeed some people in the industry trade off this strategy.

Figure 2: This is graphic illustration for Backwardation



3.4 Interest Rate Swap

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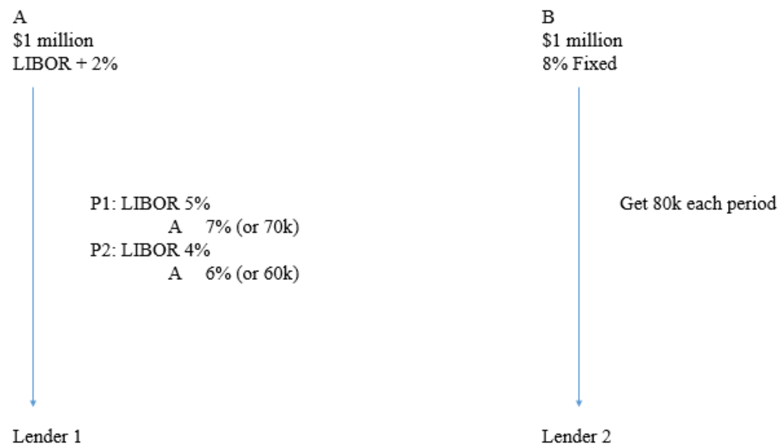
Interest Rate Swap is a very important concept through the 80s and the 90s. It created the massive boom after "911 Event" and it is a crucial concept to help us understand "08 Financial Crisis".

Swap means an action of exchange so we imagine two parties A and B as the following. A has a million and B also has a million. The value in A's hand is quoted with LIBOR + 2% and the value in B's hand is quoted with just 8%. This means that A entered a contract paying floating (not-fixed) interest rate. LIBOR refers to London Inter-Bank Offering Rates. It is a weighted average of all of the interest rates used by the big banks in London. LIBOR is computed through time and is constantly changing. That is the reason why LIBOR + 2% is a floating rate. On the other hand, 8% is just 8% and it is not changing.

The concept of swap refers to an action that party A and party B entering into an agreement such that B is paying A's interest rate and A is paying B's interest.

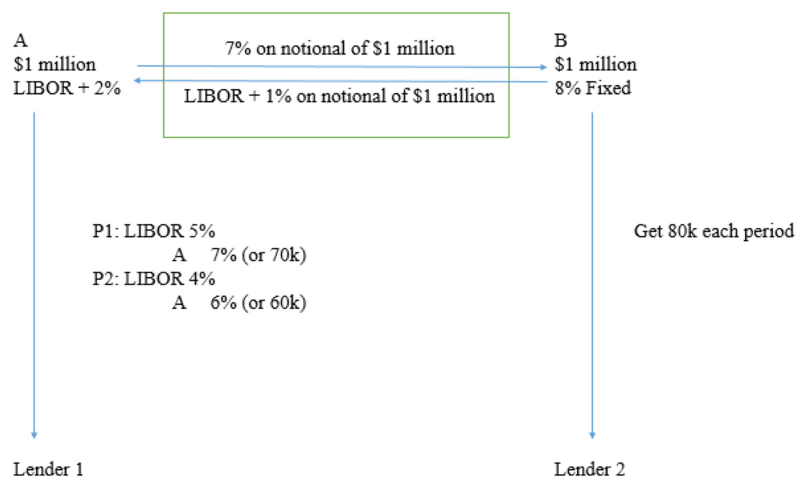
Originally, at time 1 we note LIBOR to be 5% and time 2 to be 4%. Then A will be paying \$70k for time 1 and \$60k for time 2. B is paying for \$80k for both time 1 and 2.

Figure 3: This is graphic illustration for Interest Rate Swap Step 1.



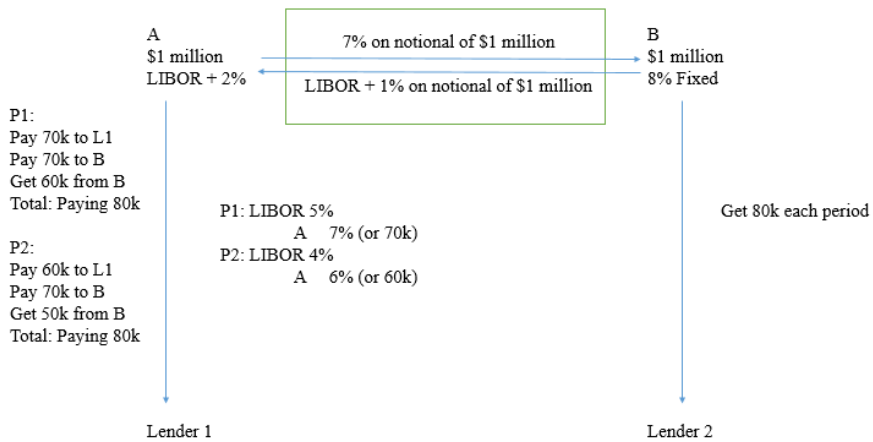
Then A found B and entered into an agreement with the following contents. A is paying B 7%. B is paying A LIBOR + 1%. The numbers are, of course, made up so that the actually equal to each other. In real life, we simply take the difference between fixed and floating rate and we exchange this difference.

Figure 4: This is graphic illustration for Interest Rate Swap Step 2.



After the swap, we look at the payoff for both parties at time 1 and time 2. At time 1, A is paying LIBOR + 2% from the original situation. Then he also pays 7% to B and received LIBOR + 1% in return. He will be paying - $70k - $70k + $60k = - $80k with is 8%, exactly the same as what B paid for originally at time 1. The same process repeats at time 2. A is paying out LIBOR + 2% which is \$60k since LIBOR is 4% at time 2. Then he pays B 7% and receives LIBOR + 1% in return. At time 2, A has payoff - \$60k - \$70k + \$50k = - \$80k which is again the same. After the swap A is paying the exact same amount every time, like B did originally with fixed rate.

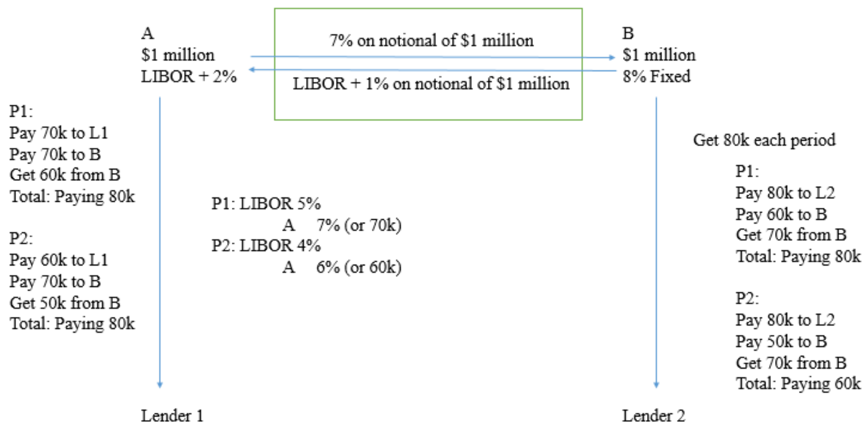
Figure 5: This is graphic illustration for Interest Rate Swap Step 3.



We can repeat the same process for party B. At time 1, B is paying 8% fixed like he did originally. Then he receives 7% from A but he needs to pay out LIBOR + 1% in return. At time 1, B has payoff - \$80k + \$70k - \$60k = - \$70k which is exactly the same as what A paid for at time 1 in the original situation. At time 2, B is paying out 8% as usual. Then he receives payment from A and pays a floating rate to A in return. His payoff, at time 2, will be - \$80k + \$70k - \$50k = - \$60k which is exactly what A paid for at time 2 originally.

Thus, it is apparent that the rate swapped between A and B. This is called interest rate swap.

Figure 6: This is graphic illustration for Interest Rate Swap Step 4.



3.5 Credit Default Swap

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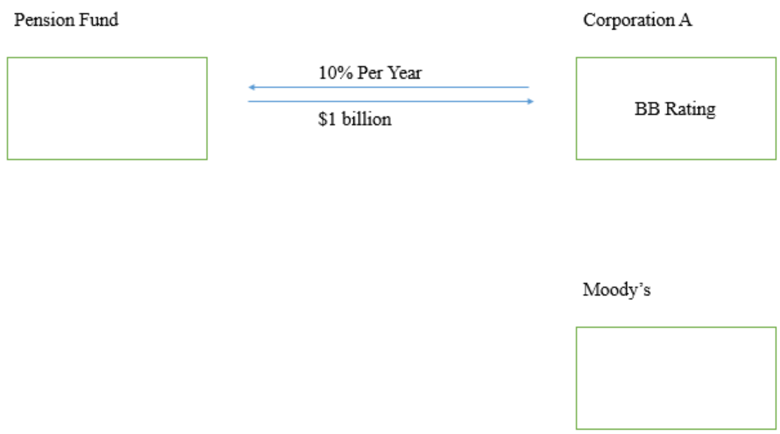
Credit Default Swap is a similar concept as Interest Rate Swap. Instead of exchanging interest rate, this action is exchanging credit debt. The reason that this is happening is mainly because people want to take on more risky assets in their book to achieve higher return.

Let us imagine a pension fund managing retirement fund for American people. They are not allowed to trade stocks so they are only looking at highly rated corporation debts (or bonds) to invest. Then there is a corporation A in this market. He is rated BB by Moody's (a rating agency). Originally pension fund can simply buy the corporation bond from A. The action requires pension fund to pay, for example, $1 billion up front and then to collect 10% annual return for a certain amount of years, and finally receive original payment in the end (which is exactly how bond works).

Figure 7: This is graphic illustration for Credit Default Swap Step 1.

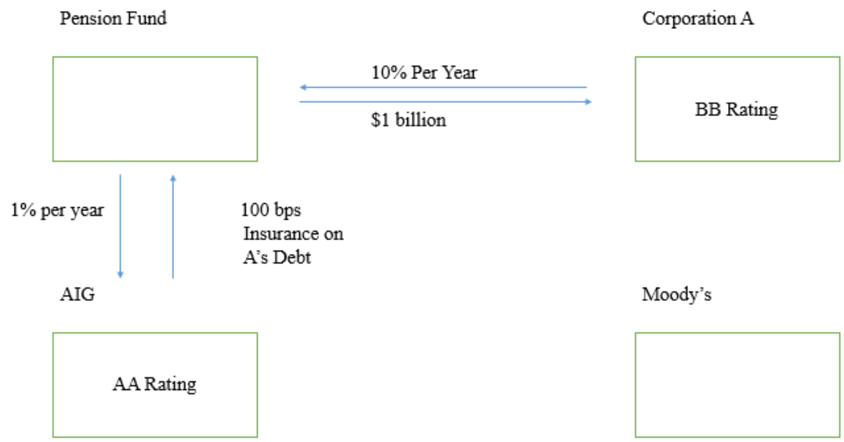


However, if Moody’s rate corporation A to be BB (lower than AA), then pension fund may not be allowed to invest in A. Then this is where insurance company, like AIG (American International Group), comes in to help to make this investment happen (even though pension fund is not allowed to buy). AIG is rated AA and AIG can sign a contract with pension fund to secure the purchase of corporation A’s bond investment. With this contract signed, the investment on A does not seem too bad anymore. In the industry, this practice will increase the rating of A so that corporation A is no more rated as BB but probably AA.

AIG could sign a contract with pension fund with 1% a year annual rate (from pension fund) and an insurance on A’s debt with 100 bps (1 basis point, aka 1bp, is $0.01 \times 0.01 = 0.0001$) of the upfront payment, $1 billion. Then if corporation A defaults in the future, AIG is responsible for the $1 billion dollars for the pension fund. Hence, pension fund has “no” risk.

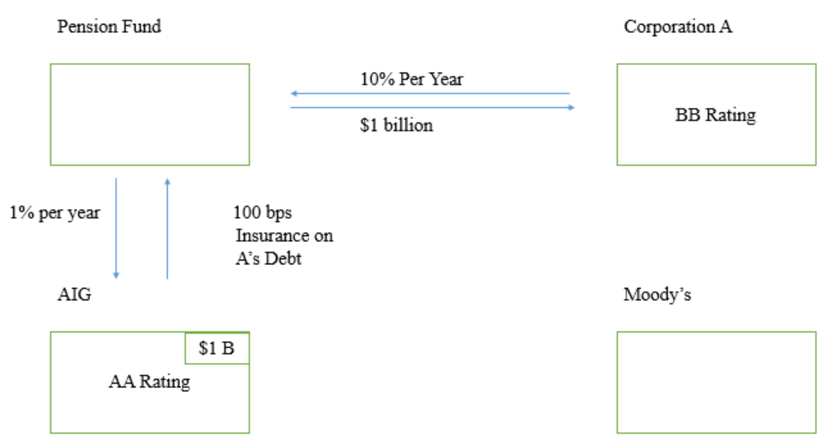
This seems like a perfect situation. All of the parties are happy. Pension fund can make more seemingly-safe investment with higher interest rate so pension fund is happy. Corporation A is also happy because even though they are rated BB they still get enough money they would have not received with BB rating. AIG is happy because they are simply sitting at home waiting to collect premiums from pension fund. The probability that corporation A defaults, in practice, is very small. In real life, it is indeed very small. These companies seldom default; but when they do default, they default at the same time!

Figure 8: This is graphic illustration for Credit Default Swap Step 2.



The contract is signed in a way that AIG should pay back $1 billion if A defaults. However, the contract says no "frozen" cash whatsoever for AIG. In practice, AIG is taking on a huge amount of potentially defaulted credits and computing an average or a weighted average based on probability the defaulting event could happen and simply leave a percentage of $1 billion on its book. Theoretically, we think in a way that AIG should have frozen $1 billion in its account and never touch that $1 billion unless all of the stories between pension fund and the corporation A are over (meaning all of the bonds expired). In practice, they do not do this. Instead, AIG would take a probability of the event A defaults and simply freezes that percentage of $1 billion on its account. Sine in real life companies seldom defaults, AIG only needs to freeze a very small amount of $1 billion.

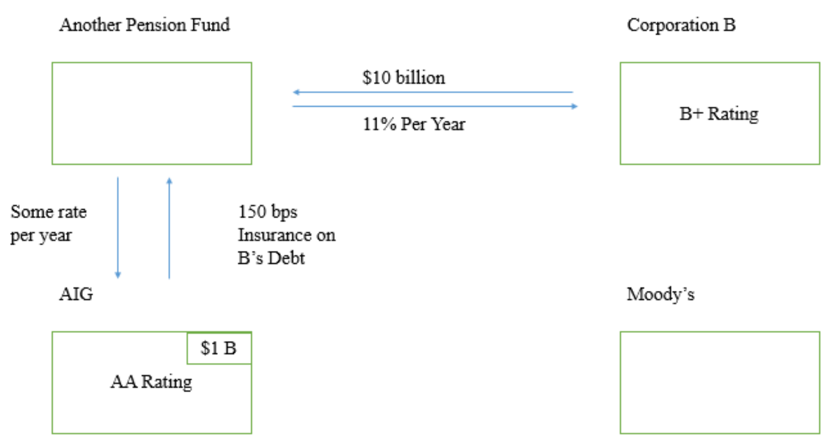
Figure 9: This is graphic illustration for Credit Default Swap Step 3.



This situation is just so "perfect" for every party in this market. It is too perfect to miss. Thus a lot of people would want to repeat this action infinitely amount so that they can all make money with no risks forever. The situation does not work this way!

In practice, AIG can insure other companies like corporation C with pension B. Then they can repeat this action again and again until one collapses. As long as one collapses, AIG needs to start to pay for default since it is signed up in the contract. If not, they will get sued. Once a corporation defaults, AIG will be downgraded by Moody's. Then the seemingly-safe companies are no more safe anymore since AIG is now downgraded. Then more and more corporation will start to default, causing more and more pension funds to ask AIG for money back. AIG only froze a certain percentage of the money and simply can't pay it back. That is why AIG is in the danger of bankruptcy.

Figure 10: This is graphic illustration for Credit Default Swap Step 1.



Why is this important? I personally do not recommend retail investors to trade bonds or any sort of treasuries securities. For the number of assets retail investors are dealing with, the profit gained from bonds and treasury securities is probably not big enough and does not worth your time. Is it safe? In textbook, yes. These securities, in textbook, are called risk-free investments. In real life, they are not. They are safe for 10 years and when they default they default all together at the same time, burning everything you ever made. The reason to understand all of these transactions is to help us understand what is truly going on in real life. In financial sector, there is no way you do not consider an investment in AIG. AIG is one of the big cap stocks in financial sector and one should always keep an eye on these big companies. These companies will mark your portfolio to a market-like performance and hence give you a very ideal risk-return profile. However, it is not to say that these companies worth long-term investment. Despite the critics above, this is not to say that fixed income and derivatives are not important. They are fairly important in dealing with ultra-high net worth assets management. In the amount of above a billion dollars, portfolios are usually hedged with some sort of derivatives products in fixed income and it is often fairly important to understand how the factors discussed in this chapter affect portfolio.

4 §Fundamentals§

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4.1 Valuation

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This part we will be talking about the values of a security, which is quoted by Warren Buffett in multiple occasions as the intrinsic values. The concept of intrinsic value is developed by Benjamin Graham and along the way a system of investment strategy investing in the true value of securities are built. We are all familiar with the value investing story, but we may not be that familiar about Poincare. Poincare was a French mathematician and physicist born in the 1850s and he was the first one introduced geometrically the intrinsic point of view concept. One of his famous models, the Poincar disk model, represents points in the hyperbolic plain as points in the interior of a Euclidean circle. In this model, lines are not straight as we used to see. Instead, lines are represented by areas of circles that are orthogonal to the circle defining the disk. Such models unveil that there may be one or more intersections in reality, although we only see one.

This concept, in my experience, happens in stock market every day. This is also what triggers majority of buyers and sellers to participate in this market. Then there is a corollary argument after this. The Poincare Disk Model differentiates intrinsic and extrinsic value. That is to say the market is exhibiting us a price that may or may not be its true value. Isn't this against Efficient Market Hypothesis? Not really. The Efficient Market Hypothesis is defined in a way that the price is constantly changing by a discovery of a new anomaly. We had utility curve by Moskowitz so that we have a market premium factor. We also know Benjamin Graham introduced value investing strategy so we added in value-minus-growth factor to determine whether a stock is a value stock or growth stocks. In 2013, we have Novy-Marx introduced Gross Profitability, a new factor that completely destroyed the conventional Fama-French 3-factor model. This will keep going and the expected return of security prices on the left hand side of the equation is constantly being updated. If the leading economists and researchers can advance our pricing models constantly as fast as the market variants, then maybe we do have a changing stock market Poincare Disk and we can constantly grab the other side of value we do not see. If the leading scientists cannot or for some period of time fail to do that, then market is at that time inefficient.

This goes back to an original argument I was making about Efficient Market Hypothesis. The market tends to be more efficient when time goes on and assets under management increases to infinity. Making one million dollars is probably a lot easier than making one billion dollars. Making one million dollars in a year is probably a lot easier than making one billion dollars in a year.

Finally, I would like to finish this part by quoting one of my favorite professors' words, "market is mostly efficient, it is like finding a hundred dollar bill on the street; it does occur, but it occurs not that often."

4.2 Discounted Cash Flow (DCF) Valuation

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In the primary market especially before an Initial Public Offering or a Mergers and Acquisition deal, financial analysts on buy side or sell side are responsible to provide all sorts of analysis on valuing the security prices. DCF Valuation is the most common of them all. Valuation is the process by which forecasts of future performance are converted into estimates of the value of the firm (enterprise value). The free cash flow for common equity, FCF_E, goes the following form.

$$FCF_E = Cash\ From\ Operations$$
$$-\triangle Operating\ Cash + Cash\ From\ Investing$$
$$+\triangle Debt - Dividends\ Paid\ on\ Preferred\ stock + \triangle PreferredStock$$

The valuation is a sum of present value of future cash inflow discounted by a factor, r. The equation takes the following form.

$$V_0 = \sum_{t=1}^{\infty} \frac{FCF_{Et}}{(1+r)^t}$$

This states that the true valuation of this company is the sum of the stream of cash flows from the future discounted to present value. This is the cash flow after meeting all cash requirements such as working capital funding, capital expenditures, debt payments, preferred stock dividends, and so on, which is in the end "free" and distributed to common shareholders. Hence this valuation method is called DCF Valuation for common equity value.

The above equation is infinite series. Mathematically speaking this will take the valuation to infinity which is unlikely to be true. Then we need to subjectively pick forecasting horizon based on either experience or clients' requirements. That is, we are making an assumption that the valuation company in the model will hit constant growth rate starting from sometime in the future. We are assuming at some time this company will enter into a steady state and will have constant sales growth rate for then on. The year in which the steady state starts is called terminal year, and the value for that year is called terminal value. Then we need to discount the terminal value to get the present value. After that we obtain a new formula for the company's vale and it takes the following form.

$$V_0 = \sum_{t=1}^{T-1} \frac{FCF_{Et}}{(1+t)^t} + \frac{\frac{FCF_{Et}}{(r-g)}}{(1+r)^{T-1}}$$

The discount factor, r, can be cost of equity or weighted average cost of capital (WACC). The terminal value is calculated by assuming the the value in a perpetuity. This is a series that will be received forever with cost of equity, r, and growth rate, g. We assumed that the cost of equity is larger than growth rate in this formula.

For the cost of equity, we can use Capital Asset Pricing Model (CAPM) to calculated return on equity (cost of equity, required rate of equity, and etc.). Let cost of equity to be r_e, risk-free rate of return to be r_f, systemic risk to be beta β, and equity risk premium to be $r_m - r_f$ while r_m is market return. The formula takes the following form.

$$r_e = r_f + \beta \times (r_m - r_f)$$

4.3 Residual Income Valuation

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The next method that is also widely used is Residual Income Valuation. Under the same assumptions, this valuation methodology will produce the exact same results as DCF Valuation.

As opposed to DCF Valuation which calculates cash flows directly, residual income models "value" accounting numbers. Although the residual income models do not "value" accounting numbers, they are a way to perform valuation by focusing on accounting measures and drivers of value creation, which is the future residual income and future abnormal rates of return on equity. This is not a methodology superior to DCF, but rather it is a framework to gain deeper and keener insight of how the forecasted future performance underlying a DCF valuation is getting transformed into a value estimate. Residual Income Valuation methodology allows us to see and to appreciate these future performances such as future ROEs with incredible clarity.

The method takes the value of a firm's common equity at the valuation date, v_0, as a function of the firm's book value of common equity, BVE_t, on the valuation date plus the present value (PV) of the firm's future residual income (RI) to common equity holders discounted at the firm's cost of equity capital, r. The equation takes the following form.

$$V_0 = BVE_0 + \sum_{t=1}^{\infty} \frac{NI_t - (r \times BVE_{t-1})}{(1+r)^t}$$

$$V_0 = BVE_0 + \frac{NI_1 - (r \times BVE_0)}{(1+r)^1} + \frac{NI_2 - (r \times BVE_1)}{(1+r)^2} + \frac{NI_3 - (r \times BVE_2)}{(1+r)^3} + ...$$

The intuition underlying the residual income valuation is reasonably straightforward. The value of common shareholders' equity is equal to the book value of common equity (BVE), plus the present value (PV) of all expected future residual incomes. Required earnings (also known as normal earnings) equal the produce of the firm's cost of equity capital, r, and the book value of common equity at the beginning of each period, that is, $r \times BVE_{t-1}$. Abnormal earnings each period is measured by $NI_t - (r \times BVE_{t-1})$. The future residual income is the amount by which expected future earnings exceeds required earnings in all future periods. This is to state the amount of common shareholder wealth created, destroyed or maintained at a given time period. If the residual income is greater than zero in a given year, common equity value is created. If the residual income is less than zero in a given year, common equity value is destroyed. If the residual income is equal to zero, the firm has just earned the cost of common equity no more or less.

Again, we are looking an infinite series formula above and we need to subjectively assume a forecasting horizon for terminal year, exactly like we did for DCF Valuation. The formula for Residual Income Valuation takes the following form.

$$V_0 = BVE_0 + \sum_{t=1}^{T-1} \frac{NI_t - r \times BVE_{t-1}}{(1+r)^t} + \frac{\frac{NI_T - r \times BVE_{T-1}}{r-g}}{(1+r)^{T-1}}$$

Since $RI_t = NI_T - r \times BVE_{T-1}$, we have the following simplified formula.

$$V_0 = BVE_0 + \sum_{t=1}^{T-1} \frac{RI_t}{(1+r)^t} + \frac{\frac{RI_t}{r-g}}{(1+r)^{T-1}}$$

The Residual Income Valuation methodology calculates the value of common equity by the following metrics: 1) the book value of common shareholders' equity at the valuation date, 2) the present value of residual income over the explicit forecast horizon, and 3) the terminal value of residual income which is based on the present value of residual income as a growing perpetuity where growth begins in year T.

4.4 Multiples Valuation

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Besides DCF Valuation and Residual Income Valuation methodologies described above, analysts can also seek some basic understanding of enterprise alum by using multiples from the industry or other competitors. However, it is important to understand the advantages and disadvantages of valuations based on market multiples.

Majority of the occasions analysts favor DCF and RI valuation methodologies, however sometimes we may not have that much data available for us. Multiples valuation is quick and simple. It is even popular because of its simplicity. However, simplicity and speed might have problems because the methodology tends to ignore some basic facts from fundamental factors. For example, we can use P/E ratio to analyze company values. For each comparable firm, we commute its P/E ratio and then we take the average or median to form a P/E multiple. To value the target firm, we multiple the projected EPS of that firm by the mean or median P/E multiple of the comparable firms.

Based on this model, we are not trying to explain prices. We are looking at other benchmark in the market and we simply look at a relative valuation metric instead of looking at future cash flows. It is essential to notice that the best multiples for one industry may not be the best multiples in another. Hence, to perform multiple-based valuation, it is important to identify what the industry considers the best measure of relative value. We have the following collections of multiples, for a target firm, j, and industry comparables, c,

$$\begin{array}{lrcl} EBITDA\times: & EBITDA_j \times \frac{EV_c}{EBITDA_c} & = & EV_j \\ EBIT\times: & EBIT_j \times \frac{EV_c}{EBIT_c} & = & EV_j \\ Sales\times: & Sales_j \times \frac{EV_c}{Sales_c} & = & EV_j \\ Customer\times: & Customer_j \times \frac{EV_c}{Customer_c} & = & EV_j \\ Total\ Assets\times: & Total\ Assets_j \times \frac{EV_c}{Total\ Assets_c} & = & EV_j \end{array}$$

For common equity, we have the following (note that EQ refers to *Common Equity Value*)

$$\begin{array}{lcl} Net\ Income\times: & & \\ Net\ Income_j \times \frac{Market\ Value\ of\ Common\ Equity_c}{Net\ Income_c} & = & EQ_j \\ Earnings\ Before\ Tax\times: & & \\ Earnings\ Before\ Tax_j \times \frac{Market\ Value\ of\ Common\ Equity_c}{Earnings\ Before\ Tax_c} & = & EQ_j \\ Book\ Value\times: & & \\ Book\ Value_j \times \frac{Market\ Value\ of\ Common\ Equity_c}{Book\ Value_c} & = & EQ_j \end{array}$$

For stock price, we have the following

$$\begin{array}{lcl} Price - Earnings(P/E)\times: & & \\ EPS_j \times \frac{Common\ Stock\ Price_c}{EPS_c} & = & Common\ Stock\ Price_j \\ Cash\ Flow\ to\ Common\times: & & \\ Cash\ Flow_j \times \frac{Common\ Stock\ Price_c}{Cash\ Flow\ to\ Common_c} & = & Common\ Stock\ Price_j \end{array}$$

5 §Asset Pricing§

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5.1 What is Greed?

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To explain the underlying emotions, greed and hope, in the market, we present two paths to analyze these psychological feelings. By looking at cross-sectional data, we can interpret empirical results from intrinsic and extrinsic point of view. Securities sold on the market bare certain amount of value. It is the value of how much each security worth by itself, hence it is the intrinsic value. Early literatures by Benjamin Graham discussed a lot of the fundamental indicators that we could be studying to understand intrinsic value. On the other side, market in which the securities are sold is on an auction bases. That is, traders buy and sell freely by bid and ask. This is largely due to negotiation and communication aspects between market participants. To date, we do not have any literature proving that this is completely rational. However, this does not hinder us to draw some reasonable arguments from applying the moving averages.

Based on Modigliani and Miller Theory, Yin (Nov. 2015) has examined the Market Value Balance Sheet (MVBS) from an accounting standpoint. The work, derived from MM Theory, describe a corporation from sum of Cash and Enterprise Value (EV) on the left side and sum of Debt (D) and Market Equity (ME) on the right side. The goal is to describe corporate activities with Enterprise Value (EV). Yin (Nov. 2015) pointed out that the scholars in behavioral finance and asset pricing barely used the model. Traditional cross-sectional study done by De Bondt and Thaler (1985) put a lot of attention on Long Run Reversals and in particular updating Fama-French three-factor model [11]. Their work sort of stock universe by winners and losers which is a measurement of stock returns. They are able to construct a replica portfolio and generate a market-like return by a long portfolio in long-run losers and a short portfolio in long-run winners. However, this sort solely depends on market returns and does not take any other fundamental factors into consideration. Hawanini and Keim (1995) attempted to sort the stock pool by the size [24]. They claim that small stocks have outperformed large stocks by about 12% a year over 1951-1989 period. This argument has been developed on stock-picking skills between buying a lot of small stocks or big stocks. The answer goes back to study the risk-return in the stock profiles. Their paper also studied the stock universe by market-to-book sort, yet book value does a poor job of describing corporate activities. Fama and French (1993) also put a lot of effort in studying risk of book-to-market ratio [18]. On the market value side, Jegadeesh (1990) examined the cross-sectional returns by the returns of small-, medium-, and large size-quintile portfolios in time t. However, these studies do not to describe how corporate activities over time affect cross-sectional stock returns [27].

First, we take MM Theory and examine Enterprise Value by applying cross-sectional study on *compustat* stocks universe. We provide three methods to sort the stock universe. Among them, $\gamma_{i,t}$ is the most important one and it describes the change of Enterprise Value (EV) over Market Equity. The results are consistent with Piotroski (2000). We conclude an investor can construct a "greed" strategy such that he can generate alpha by buying the most "greedy" stocks and selling the least "greedy" stocks.

Next, we present a model looking at different time period persistence and momentum among cross-sectional returns. There are some literatures related to this topic. Hendricks, Patel, and Zeckhauser (1993) [25], Goetzmann and Ibbotson (1994) [20], Brown and Goetzmann (1995) [7], and Wermers (1996) [43] find evidence of persistence in mutual fund performance over short-term horizons of one to three years. Grinblatt and Titman (1989) [21], Elton, Gruber, Das, and Hlavka (1993) [15], and Elton, Gruber, Das, and Blake (1996) [14] study mutual fund return predictability over longer horizons of five to ten years, and attribute this to manager stock picking skills. Jensen (1969), however, presents contrary evidence and explains that good subsequent performance follows good past performance [29]. Carhart (1992) [8] [9] explains that it is persistence in expense ratios that drive a lot of the long-term persistence in mutual fund performance. We are using moving averages, both Simple Moving Average $SMA(i,t)_n$ and Exponential Moving Average $EMA(i,t)_n$, to measure the stock returns.

5.2 Empirical Study On Greed

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From Market Value Balance Sheet (MVBS) model, we look at corporate activities (mergers and acquisitions, etc.) from market value of assets and market value of liabilities. The left side of the balance sheet is composed of Cash and Enterprise Value (EV) while the right side of the balance sheet is composed of Debt and Market Value of Equity, which has the following form:

$$MVA = MVL, \tag{12}$$

where the model denotes market value of assets on the left to be equivalent as market value of liabilities on the right. In detail, each component summarizes the following:

$$\begin{cases} C + EV &= MVA \\ D + E &= MVL \end{cases}$$

We can take equation $MVA = MVL$ and subtract Cash (C) from both sides. Then we have the following equation:

$$EV = D + E - C \tag{13}$$

For the definition of Enterprise Value (EV), Yin (Nov. 2015) assumed that market value of debt (MVD) was Total Liabilities and used the book definition for consistency purpose: $EV = Total\ Liabilities + Total\ Shareholders'\ Equity - Cash$. This definition is consistent in which each of the component comes from accounting measures. For each asset i, Yin (Nov. 2015) sort the stock universe by 1) percentile of how much Enterprise Value (EV) in Total Assets, and 2) change of Enterprise Value (EV) over Total Assets. Both definitions write the following form,

$$\omega_i = \frac{EV_i}{TA_i}, \tag{14}$$

and

$$\phi_i = \frac{\triangle EV_i}{TA_i} = \frac{EV_i - lag(EV_i, 12)}{TA_i}. \tag{15}$$

However, this definition does not include anything about market value aspect of the corporations, which we found to be lack of persuasive power. Hence, we redefine Enterprise Value (EV) in this paper as the following,

$$EV = Total\ Liabilities + Market\ Equity - Cash. \tag{16}$$

In this paper, we use this definition for Enterprise Value and we look at corporation activities from sum of Total Liabilities and Market Equity subtracted Cash of a certain firm. For for any asset i over t-period, we can rewrite equation (6) in the following form,

$$EV_{i,t} = TL_{i,t} + ME_{i,t} - C_{i,t}. \tag{17}$$

However, EV_i does not reflect anything about how certain changes may have happened for a company. By using function $lag(x, t)$, we calculate x shifts t period(s) into the past, $lag(x, t) = x_{t-1}$. We take 1st order of Enterprise Value (EV). We denote this as growth rate δ and the definition writes the following form,

$$\delta EV_{i,t} = \frac{EV_{i,t} - lag(EV_{i,t}, t)}{lag(EV_{i,t}, t)}, \tag{18}$$

and we also take the 2nd order of Enterprise Value (EV). We take the change of growth rate δEV for any asset i over t-period. The definition writes the following form,

$$\delta(\delta EV_{i,t})_{i,t} = \frac{(\delta EV_{i,t}) - lag((\delta EV_{i,t}), t)}{lag((\delta EV_{i,t}), t)}, \tag{19}$$

Fama and French (1993), De Bondt and Thayler (1985), and Jegadeesh (1990) all presented models implying the importance of book-to-market factor. To really show how changes of corporate activities and market equity affect cross-sectional stock returns, we have the following definition,

$$\gamma_{i,t} = \frac{\triangle EV_{i,t}}{ME_{i,t}} = \frac{EV_{i,t} - lag(EV_{i,t}, t)}{lag(ME_{i,t}, t)}, \tag{20}$$

Moving Averages are used by chartist in practice. To date, there has not been any successful literatures document the advantage of trading with moving averages.

Moving Averages can be simple or exponentially. Simple Moving Average (SMA) records a smoother path than observed population where as Exponential Moving Average (EMA) puts more weight as time goes on on Simple Moving Average. For each observed price, we can calculate simple moving averages, as below

$$SMA(i,t)_n = \frac{1}{n}\sum_{j=1}^{n} P(i,t)_j, \tag{21}$$

so far each stock i at time t we can calculate $n-day$ simple moving averages by taking the sum of the n days observed prices and divided by n. We can apply this to data of cross-sectional returns. We first calculate take the first derivative of observable prices in the market in the following form. That is the return of stock price, $\delta P_{i,t}$, in the following form,

$$\delta P_{i,t} = \frac{\triangle P_{i,t}}{P_{i,t-1}} = \frac{P_{i,t} - lag(P_{i,t}, t)}{lag(P_{i,t}, t)}, \tag{22}$$

and then we apply simple moving averages to the return. We have,

$$SMA(\delta P_{i,t})_n = \frac{1}{n}\sum_{j=1}^{n} (\delta P_{i,t})_j, \tag{23}$$

and this allows us to capture a slower moving time-series of price actions. We can sort cross-sectional stock returns by $SMA(\delta P_{i,t})_n$, outperforming FF-3 replica.

Moreover, we can look at exponential moving averages (EMA). We define exponential moving average as a weighted moving averages by taking a weight distributed in perspective of time between observed data and simple moving averages of observed data. The concept of exponential comes from the fact that the formula is constructed such that the weight on the simple moving averages increases when the time lags longer into the past. There is no particular reason we have this definition. One can do the opposite and still achieve the same arguments. The formula grows the following form,

$$EMA(i,t)_n = P_{i,t}\theta_n + SMA(i,t)_n(1-\theta_n), \tag{24}$$

with $\theta_n = 1/(n+1)$ to be a weight calculated exponentially. Hence, equation (15) can also be extended to the following form,

$$EMA(i,t)_n = P_{i,t}\frac{1}{(n+1)} + \frac{1}{n}\sum_{j=1}^{n} P(i,t)_j(1 - \frac{1}{(n+1)}). \tag{25}$$

Next, we apply exponential moving averages to return of stock prices, $\delta P_{i,t}$, and we will have the following formula,

$$EMA(\delta P_{i,t})_n = (\delta P_{i,t})\frac{1}{(n+1)} + \frac{1}{n}\sum_{j=1}^{n} (\delta P_{i,t})_j(1 - \frac{1}{(n+1)}). \tag{26}$$

Next, we apply Fama-French 3-factor model to test this idea. We apply the three definitions above to sort the *compustat* stock universe. We divide the stock universe into five quintiles and sort by each of the definition above with different time periods. Then we examine the return by market factor, size, and book-to-market.

The FF-3 in our test has the following form:

$$r_{i,t} = \beta^i_{MKT} MKT_t + \beta^i_{SMB} SMB_t + \beta^i_{HML} HML_t + \epsilon_{i,t}, \tag{27}$$

5.3 Cross-section Study

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We conduct cross-section study to examine Enterprise Value (EV) and the change of Enterprise Value (EV) based on Yin (Nov. 2015). First, we sort the stock universe by the growth rate $\delta EV_{i,t}$. Then we sort the universe by $\gamma_{i,t}$ based on equation 10. Last we sort the universe by $\delta(\delta EV_{i,t})$. We conduct cross-section regression to study the concept Enterprise Value (EV) in the MVBS model. Yin (Nov. 2015) used both equal-weight and value-weight. This paper we drop equal-weight because it is too far away from the practice and this method does not present results close to the industry numbers. All portfolios are under value-weight for each sort. We conducted all studies with FF-3 and we drop Carhart 4-factor model which was originally applied in Yin (Nov. 2015).

Yin (Nov. 2015) have shown that all sorts present excess return of large portfolio than small portfolio to be a positive percentile. In other words, an investor could construct a portfolio simply by a long portfolio of high Enterprise Value (EV) stocks (or big increase on Enterprise Value) and a short portfolio of low Enterprise Value (EV) stocks (or small increase on Enterprise Value) blindly with NYSE breaks and he could make an average of 0.502% (from "XRet" in Table (1)). This results of a universally positive excess return from the large Enterprise Value (EV) portfolio minus the small Enterprise Value (EV) portfolio show consistency with traditional FF-3 sorted by market cap, implying the same trading strategy and investment philosophy between an investor using book-to-market and an investor using Enterprise Value (EV). As Table (1) shown, a strategy by FF-3 can be replicated with -0.057, -0.137, and 0.529 loadings on market, size, and book-to-market factors. The extremely significant positive loading on book-to-market ratio shows that this is a value strategy. The investment decisions betting on expected increase on corporate activities is consistent with value investing in the efficient market. We find further consistent results as Yin (Nov. 2015).

Next, we conduct cross-section study to examine how different sorts among definitions derived from moving averages affect alphas throughout different time period. We have four panels presented in Table (4). We arbitrarily choose time period to be 10-, 20-, 30-, 40-, and 50-month to run time-series regressions. We put everything together to form panel depending on the sort. There are 14 out of 20 t-stat in the table that are greater than 1.96, which are statistically significant.

Last, we conclude the trading strategies from all of the examinations.

This sub-section we sort the universe by $\delta EV_{i,t}$. For each asset i, we calculate the change of $EV_{i,t}$ based on different time t-period. We select 1-, 2-, 3-, 4-, and 5- year in the past. For t-period, we sort stock universe by $\delta EV_{i,t}$ to ten deciles and we examine each portfolio with FF-3 model. In Table (3) Panel A, we present the access return of each L/S strategy for each t-period in every sort.

We observe that access return increases as time lags longer into the past. For each sort, access return is calculated from the difference of the winning portfolio and the losing portfolio for each of the t-period. For example, we sort the universe by by the growth rate $\delta EV_{i,12}$ into ten deciles and we take the difference of top decile and bottom decile, 0.172% monthly return with t-stat to be 1.06. This strategy beats FF-3 benchmark by 0.394% with t-stat at 2.48. The significant t-stat on alpha shows us that an investor can simply buy the companies with large growth rate $\delta EV_{i,12}$ in the past year and sell the companies with small growth rate $\delta EV_{i,12}$ in the same period to beat the market.

This strategy turns out to be the same as large-cap growth stocks strategy. Table (4) shows us a significant negative loading on HML factor for this strategy. There is a 0.511 loading tilted to growth stocks more than value stocks with a t-stat of 8.47. For each of the period 12-, 24-, 36-, 48-, and 60-month, we have loadings on book-to-market ratio to be -0.511, 0.673, 0.696, 0.810, and 0.844 with corresponding t-stat to be -8.47, 11.63, 12.78, 16.75, and 18.21. We see monotonicity between t-period and loadings on book-to-market ratio as well as corresponding t-stat. This is consistent with our expectation since equation 7 tells us that an increase on $ME_{i,t}$ will result in an increase of $EV_{i,t}$ as well. This means the longer in the past we study Enterprise Value (EV) changes the more weight we would be putting on value stocks.

5.4 Sort and Analyze

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This sub-section we sort the universe by $\gamma_{i,t}$, which is the percentage of the increase on Enterprise Value (EV) for an asset i during t-period over the $ME_{i,t}$ of that asset during the same period. This modification is to mimic the book-to-market ratio. For high book-to-market ratio stocks, we call this tilt or a positive loading on B/M factor a value strategy. If an investor is targeting the stocks that have significant growth of Enterprise Value (EV), the investor is really trying to invest in the value of the company, which is high book-to-market ratio stocks. On one hand, EV is calculated from discounting the net cash flow of the company. On the other hand, EV is also increasing if there is a significant buyer in the market trying to push the price up, hence increasing market value of equity $ME_{i,t}$. This type of sort is really trying to discovering EV changes corresponding to market value of equity.

From Table (3) Panel B, we present, in each row, the L/S strategy on $\gamma_{i,t}$ sort during different t-period. The apparent results show us a monotonicity between t-period and t-stat of alpha. The The alpha of each sort for each t-period increases as the time t-period lags longer into the past. We present 12-, 24-, 36-, 48-, and 60-period lagged into the past. That is to say for each of these periods, we have increasing t-stat on access return and t-stat on alpha. This shows us that as times lags longer into the past an investor will be making higher expected return on the portfolios and the results are more and more likely to happen.

This part we also find consistent results as we sort by $\delta EV_{i,t}$ presented in Section 4.2. That is to say, an investor is investing in value stocks when he targets long-term Enterprise Value (EV) growth. This is true disregard if he is comparing the growth to Enterprise Value or Market Equity.

We sort the universe by $\delta(\delta EV_{i,t})$. In Table (3) Panel C, we do not observe any monotonicity between any two parameters. The t-stat are relatively random compare to those in other panels. The highest t-stat is 1.78 for $\delta(\delta EV_{i,12})$ and this gives an alpha of 0.107 with insignificant t-stat at 1.23. We conclude that this model failed to show us any regular changes over time on the second order of Enterprise Value (EV).

Although not apparent, the value strategy does appear in Table (4) Panel C as we expected. Panel C shows us relatively increasing loadings from -0.118 to 0.211 with significant t-stat on HML factor. The recent, 12-month, sort would put more weight growth stocks. As time lags longer into the past, the loadings turn positive and gradually increasing. This is consistent with all the findings above in Section 4.2 and Section 4.3.

We sort the stock universe by simple moving averages in Panel D, Table (4). The results are universally the same disregard the time-period in the model. The excess return has a t-statistic at 1.72 and the α has a t-statistic of 1.08. Although the results not statistically significant, they tell us that for Simple Moving Average trading strategy is not dependent on the difference of time frame. This result does not surprise us since Simple Moving Average is generated directly from cross-sectional stock returns.

We sort the stock universe by exponential moving averages in Panel E, Table (4). The sort by EMA gives us a relatively monotonic excess monthly return, ranging from 0.344% to 0.528%. The t-statistics for all of the excess returns are above 1.96. Moreover, we achieved significantly positive alphas for all five time periods. For an investor who trade by 10-month period, he could mandatorily buy portfolios performed poorly in 10-month and sell portfolios performed well in 10-month and create an alpha of 0.461% monthly excess returns than FF-3.

Based on the construction of EMA, we know that the definition would put more weight on SMA as time lags longer into the past. That is, an investor could change different time period to achieve similar strategy as short-run reversal and long-run reversal. In Panel E, we observe that 40-month period is relatively insignificant compared to the other periods. The second significant period is 50-month. This is equivalent to the mechanism as if an investor is running long-run reversal strategy.

We sort the stock universe by difference between stock returns and SMA in Panel F, Table (4). That is, we are looking at how much bigger the return of stock universe is compared to simple moving averages for each period. By taking different arbitrary number of time periods into consideration, we are looking at a trading strategy targeting how far away the price is deviating from moving averages.

We can use Graph (1) as an assistance for us. Graph (1) is projected as the price and simple moving averages on prices. This is a time-series plot before taking derivatives. Our model is constructed by looking at the derivatives of these plots. For each line (plot), there will be an imagined tangent as time changes, which is the first order derivative.

In other words, we are looking at how fast the price grows higher than simple moving averages. For each time period t, the model is simulating an investor mandatorily holding a portfolio with a long position in stocks that have similar returns in price and SMA and a short position in stocks that have big difference in price and SMA. Intuitively, this is almost similar as balance your portfolio when it is close to moving averages or is crossing over moving averages and sell when it is far away from moving averages.

We sort the stock universe by difference between stock returns and EMA in Panel G, Table (4). This strategy is mechanically similar as sorting by $\delta P_{i,t} - SMA(\delta P_{i,t})_n$, yet there is a big difference between moving averages. By equation (22), (25), and (26), the EMA puts more weight on SMA than return of price as time period lags longer into the past. That is, we expect EMA to look more like SMA when time lags longer but to look more like return of price when time lags shorter.

The data presented in Table (4) show us all the t-statistics for different periods to be the same between sorting by $\delta P_{i,t} - SMA(\delta P_{i,t})_n$ and sorting by $\delta P_{i,t} - EMA(\delta P_{i,t})_n$ except for t to be 40-month. However, we observe that t-statistic for 40-month in Panel G is 2.58 which is bigger than that of Panel F at 2.57. The little detail show us that if we expand the time period and lag the period longer in the past we would have achieved some more significant results.

The results from cross-sectional stock returns showed us we can generate an alpha of 0.42% monthly by trading of exponential moving averages with 10-month period, which is consistent with the result of short-run reversal.

We summarize all L/S portfolios with coefficients on market, size, and book-to-market factors in Table (4). Among three panels, A, B, and C, are the results for coefficients and corresponding t-stat for all FF-3 model replicas on $\delta EV_{i,t}$, $\gamma_{i,t}$, and $\delta(\delta EV_{i,t})$ sorts. Yin (Nov. 2015) have conducted parallel study on ω_i, and ϕ_i both lagged 12-month in the past. In the previous work by Yin (Nov. 2015), the results from his test suggested the highest Sharpe Ratio that can be constructed is the portfolio sort by the change of Enterprise value (EV) over Total Assets. Moreover, the more decile the universe is separated the more attractive the Sharpe Ratio is. His L/S replica portfolio of five breaks gives a Sharpe Ratio of 0.4299 and the L/S replica portfolio of ten breaks gives a Sharpe Ratio of 0.5511, compared to the Sharpe Ratio of the replica portfolio with traditional sort by market value to be 0.3066. Although his work generated a higher Sharpe Ratio by change of Enterprise Value (EV) over Total Assets, he argued that the Change of Enterprise Value (EV) is the key here in this model. When a company acquires an asset, it is the value added to the whole company from that acquisition that attracts and affects decision making process.

The work is doing a fine job describing how previous year Enterprise Value (EV) affecting cross-sectional stock returns at present time. However, such model provides no explanatory power for long time into the past, hence it does a poor job in forecasting ability especially further into the future. Table (2) presents the table from Yin (Nov. 2015) and we can argue selected winning strategy from Table (1) with 0.5511 Sharpe Ratio is actually a lucky event.

Table (3) we present three panels on top the foundation of Yin (Nov. 2015) and we show that investment strategy with studying the change of Enterprise Value (EV) actually gradually improves as time goes along. This pattern is extremely obvious in Panel B of Table (3) and similar patterns can be observed for other panels. To study cross-sectional stock returns, Enterprise Value (EV) and the change of Enterprise Value (EV) are two essential factors before sound investment decisions can be made.

Every time there is a merger or acquisition happen, the Enterprise Value (EV) changes. The corporate activities could be generating synergies and could also destroying synergies. By studying the change of Enterprise Value (EV), we are really trying to capture whether the corporation is aggressive or not in its expansions. If a management team or a board is participating in M&A deals one by one, it is representative that this is a very greedy and aggressive group of people managing this company. The truth behind the act of M&A deals simply states that there is underlying greed in this security. If a person does not want to grow his money, why on earth would he buy new assets and take on more risk? Hence, we are really measuring "hope" or "greed" in the underlying assets. By calculating $\delta EV_{i,t}$, $\gamma_{i,t}$, and $\delta(\delta EV_{i,t})$, we can construct portfolios that buy the companies people hope the most and sell the companies people hope the least, outperforming the market with an positive alpha. Moreover, an investor can further its certainty and achieve the results with a portfolio constructed studying longer time lagged into the past.

Table (4) present the reader four panels from technical perspective of cross-sectional stock returns. This aspect describes a series of scenarios other than Enterprise Value (EV). The prior is an extrinsic point of view and the latter an intrinsic point of view. Overall, we do observe some significant alpha in Exponential Moving Average and the difference between cross-sectional stock price and the two moving averages, $SMA(\delta P_{i,t})$ and $EMA(\delta P_{i,t})$ respectively. Panel E in Table (4) sort stock universe directly based on the Exponential Moving Average of the first order of cross-sectional stock returns. We observe significant alpha in 10-month and 50-month period, which is consistent with short- and long-run reversal strategies. Moreover, we observe relatively consistent alpha in Panel F and Panel G, which describes cross-sectional stock returns sort by $\delta P_{i,t} - SMA(\delta P_{i,t})_n$ and $\delta P_{i,t} - EMA(\delta P_{i,t})_n$. The alpha ranged from 2.57 to 2.92 consistent for these two panels and long/short strategies have shown Sharpe Ratios that are slightly above market.

6 §Price as a Martingale§

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Security prices are generally understood as random walk. Although some scholars still doubt the concept of efficient market hypothesis, there are scholars like Malkiel shown us empirical evidence that we do observe data in favor of efficient market hypothesis. He has shown us that professional investment managers do not outperform their index benchmarks and has provided us evidence that prices do digest all available information [36].

In this paper, we start with the characteristic that price follows random walk. That is, we assume market is efficient over the long run and security prices do reflect all available information out there. This is also to say that the first order derivative of price (price changes) follows binomial distribution (big probabilities that absolute values of return are small and small probabilities that absolute values of return are big). Moreover, we prove that expected return of security prices is a martingale. Samuelson (1965) advocated that price fluctuated randomly and the movement of prices can be seen as a stochastic process [41].

Jegadeesh and Titman (2007) has found evidence that strategies of buying stocks performed well in the past and selling stocks that performed poorly in the past generate significant positive returns in 3- and 12- month holding period [28]. Similar ideas are presented by De Bondt and Thaler (1985, 1987) [11][13]. This up-minus-down momentum factor in the security market provides another important ground work for us. If the strategies introduced by these scholars are worth running (which proved that they are), there would be rational investors out there who want to try these strategies. This provide liquidities for our model. If no one is willing to buy high sell low, it does not make sense for us to argue to buy low sell high. Hence, we would assume that there is some liquidities out there provided, that is, the market exists.

Another thing that we need to pay attention to is the concept of anomaly. A value strategy can be an anomaly if significant alpha can be found. A momentum strategy can be an anomaly if its alpha is significant. We define anomalies among a pool of expected returns to be the ones cannot be tolerated by an investor subjectively. This is a definition that has not been discussed before. As a matter of fact, this definition is difficult to start any quantitative approach because of the adjective "subjective". We set up this definition on purpose because we will introduce a trading frequency δ to be associated with the investor's tolerance of the security prices and hence we can start to have some discussions about anomalies.

Questions we want to answer in this paper originate from the assumptions above. If market is efficient and incorporates all publicly available information, it is rational to infer that a market grows by a various number of factors out there, say GDP, NGDP, interest rates, fundamentals, technicals, algorithms, and etc. since all of the information are public. Then we would ideally see a relatively straightened line of prices throughout the history if we compare to other indicators. However, we observed two very different variations between security market and economic market. Scholars can find factors and use pricing models to find out why the security market is so volatile; hence, they can explain the reason why we have such observations, i.e., there are a lot more spikes in the price chart of stock market than there are in GDP chart (see Figure 1 and 2). Is it all possible to simply buy the market when market is low and just let the market grow with the economy? If so, how do we do it in a consistent and rational sense that the lower the market dips the more we buy?

Figure 11: The return after interest rate with $1 investment in the market from July of 1926 to June of 2016. *Source: From Ken French Data Library.*

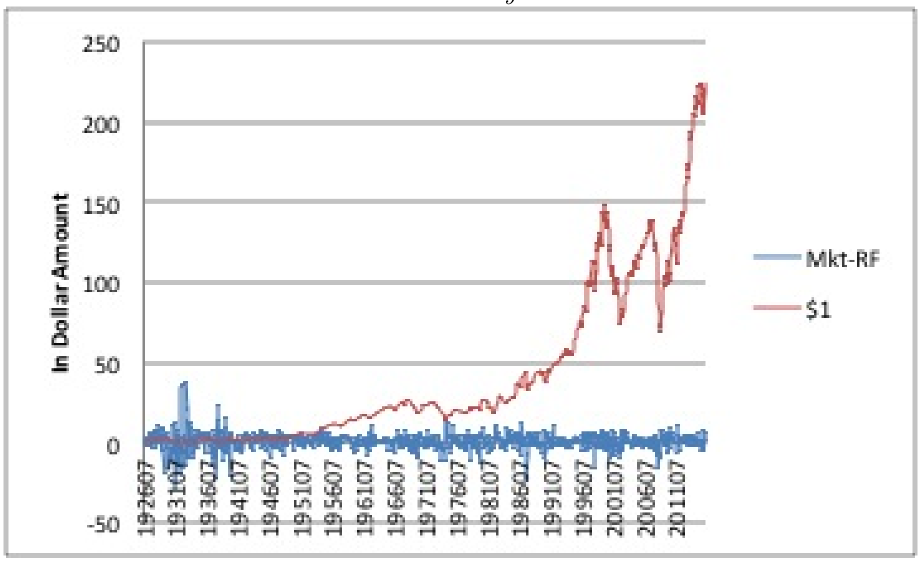
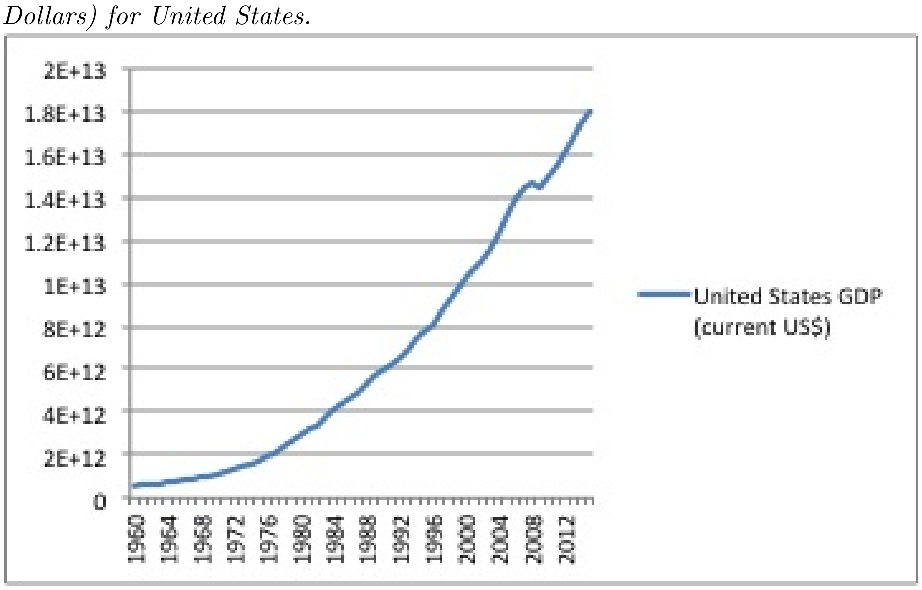


Figure 12: The GDP in US Dollars from 1960 to 2015. *Source: From World Bank Library Current GDP (in Dollars) for United States.*



At each time node n, we have some kind of information about $\omega \in \Omega$ that we know contained in a filtration, F_n. As n increases, we incorporate more information about ω, and hence ω increases. We introduce the following definition.

Definition 6.1. A discrete-time stochastic process, price, $P = \{P_t, t = 0, 1, ...\}$ is a martingale with filtration $\mathbb{F} = \{F_t, t = 0, 1, ...\} \forall t \in \mathbb{N}^+$. Moreover, the first order derivative of price, $r_t = \frac{\partial P}{\partial t}$ is also a martingale.

It is easy to check this since one needs to check that the discrete-time stochastic variable price, P_t, satisfies all the properties of a martingale (see Appendix). From the definition, we can take the first order derivative of the price and the properties would still be satisfied. We can argue that the the expected return at any time node is equal to the initially selected time node. Furthermore, we argue that the expected return for prices at time node t conditional on the filtration is the same as the one before.

Definition 6.2. Suppose that $r = \{r_t, t = 0, 1, ...\}$ is a martingale with filtration $\mathbb{F}$. Then

(i) $\mathbb{E}(r_t) = \mathbb{E}(r_0)\ \forall t \in \mathbb{N}^+$

(ii) $\mathbb{E}(r_t | F_m) = r_s, \forall r > 0, \forall s > 0,\ \textit{and}\ s < t.$

Alternatively, we can also present large dataset to show that years of historical data is plotted and can look like normal distribution. However, since data set has survival bias, we choose a theoretical approach to show you a more completed argument.

With the idea of price following martingale, we can look at the next section to discuss moving averages.

7 §Trading§

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7.1 Moving Averages

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We look at price as a time-series function. That is, for each time node t, we have an executed price P. This can be seen as a discrete function with a map from time to price. We can denote price as P_t.

Anything can happen any time in the market and an individual price level is usually very volatile. We incorporate the concept of moving average so that we can look at the average price of some period in the past, which allows us to overlook the trading noise in the market.

Definition 7.1. Simple moving average (SMA) takes the average of prices traded n time period(s) in the past,

$$SMA_n = \frac{1}{n} \sum_{t=0}^{n} P_t.$$

We can also implement the concept of exponential moving average, which is to add a certain amount of weight to the current prices when calculating the averages.

7.2 Price to Moving Averages

Based on the moving averages of the prices, we can establish the concept of price-to-moving averages.

Definition 7.2. Price-to-Moving Average is the ratio of price over a selected period of moving averages. For n periods, we have

$$P_t - SMA_n = \frac{P_t}{\frac{1}{n} \sum_{t=0}^{n} P_t} - 1$$

.

We can interpret this definition by our assumptions. We assumed that price follows random walk. The averages of a random walk should also be random walk. That is to say, we have two series of numbers following binomial distributions, which implies that the ratio should also be a binomial distribution. The data had an average expected return of 0.65%. If we imagine this to be the initial x-axis, we would observe a binomial distribution for price-to-moving averages for all three data series.

Figure 13: The graph shows returns after interest rate with $1 investment in the market along with three other moving averages with time periods to be 50-day, 100-day, and 150-day from July of 1926 to June of 2016. *Source: From Ken French Data Library.*

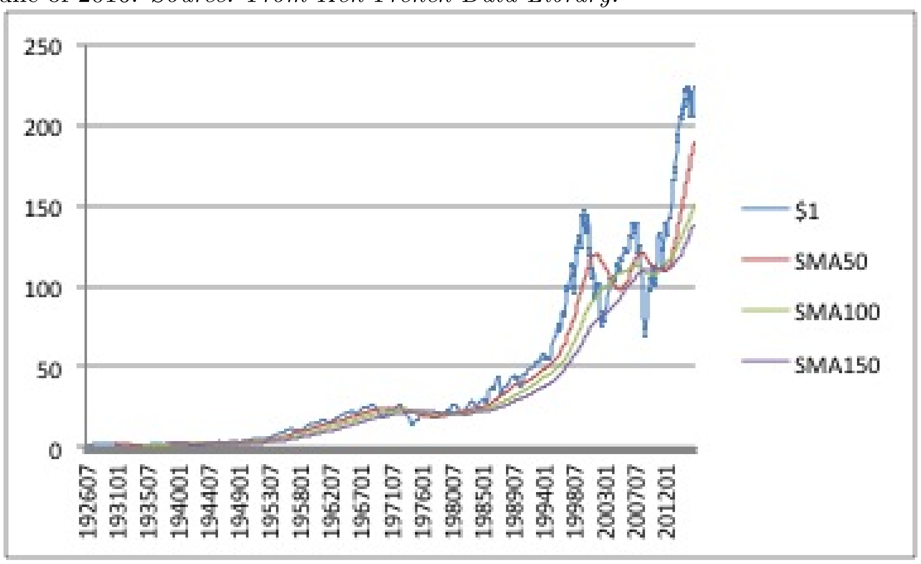
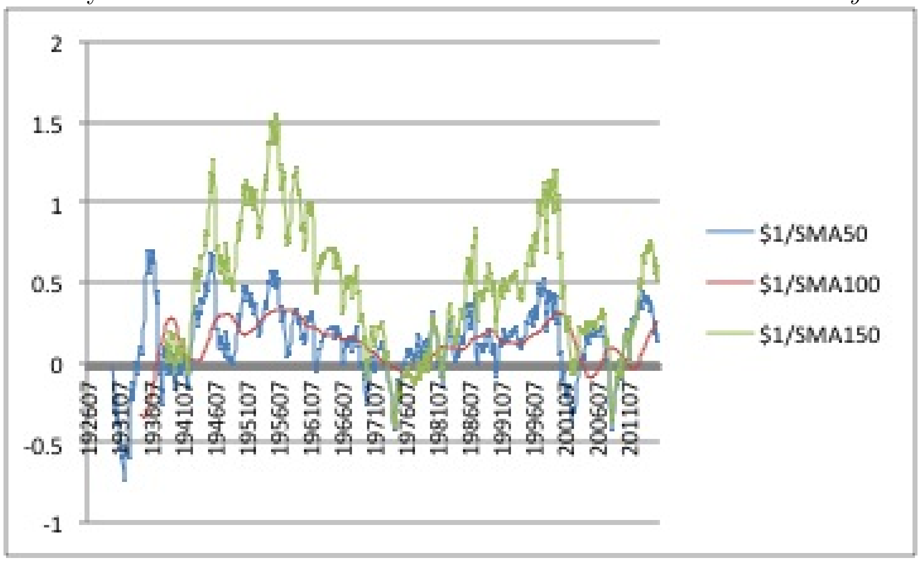


Figure 14: The graph shows price-to-moving averages with time periods to be 50-day, 100-day, and 150-day from July of 1926 to June of 2016. *Source: From Ken French Data Library.*



7.3 Trading Frequency

We defined in the introduction section of this paper that we call anomalies to be the ones among a pool of observations that cannot be tolerated by investors subjectively. This concept is associated with a trading frequency δ. An investor could be greedy or moody when he made an investment decision. We do not care. We summarize all of those emotions into δ. Emotionally impatient investors tend to be less educated and would buy very often, hence a high trading frequency (δ is big). Emotionally patient investors tend to be more educated and would buy once in a while, hence a low trading frequency (δ is small).

Based on each price-to-moving average, we can calculate standard deviation, σ. For every $p_t - SMA_n$, we have a calculated $\sigma_{t,n}$. From binomial distribution, we know that the probability that $p_t - SMA_n$ is high happens not that often whereas the probability that $p_t - SMA_n$ is low happens pretty often (i.e. bell curve). Hence, for the observation space $X = (t, n)$ and a particular event $k = (i, j)$, we have

$$\forall i, j \in \mathbb{R}, Pr(p_t - SMA_n = p_i - SMA - j) = Pr(X = k)$$

$$= \binom{n}{k} p^k (1-p)^{n-k} \forall k \in \mathbb{Z},\ where\ \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

The interpretation is the following. There exists some i and j such that the probability of the price-to-moving average equals to this particular i and j is defined by the binomial equation.

The next question we need to think about is how do we understand or even quantify how many particular pairs of (i, j) we need to choose. The answer to this question is dependent on individual investor, which is associated with trading frequency δ. Moreover, the trading frequency δ is dependent on standard deviation $\sigma_{t,n}$. In other words, we need the following definition.

Definition 7.3. With the standard deviation generated by (t', n') to be greater or equal to that generated by (t, n), the particular event k is the product of δ and total observations with (buy side) trading probability δ defined as the following

$$\exists h \in \mathbb{R} \; s.t. \; for \; (t,n) \; and \; (t',n')$$

$$\delta_t = Pr(\sigma_{t,n} \geq h\sigma_{t,n}) = \frac{\sum_{(t',n')} \mathbb{I}_{(t',n')}}{\sum_{(t,n)} \mathbb{I}_{(t,n)}}$$

with

$$\mathbb{I}_{(t',n')} = \begin{cases} 1 & , \quad when \; h\sigma_{t',n'} > \sigma_{t,n} \\ 0 & , \quad when \; h\sigma_{t',n'} \leq \sigma_{t,n} \end{cases} , \; (sign \; changes \; for \; sell \; side)$$

and

$$\mathbb{I}_{(t,n)} = \begin{cases} 1 & , \quad if \; \exists\sigma_{(t,n)} \\ 0 & , \quad otherwise \end{cases}$$

while

$$\sigma_{t,n} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}((p_t - SMA_i) - \frac{1}{n}\sum_{i=1}^{n}(p_t - SMA_i))^2}$$

This equation looks complicated, yet it is easy to understand. The amount of event k happens in a way that is dependent on trading frequency δ. Trading frequency δ is defined as a probability, which is a probability calculated to be the amount of times standard deviation of price-to-moving averages to be greater or equal than an ideal amount of times of all observations. The indicator, $\mathbb{I}$, is to pick out the amount of times that the calculated standard deviation satisfies condition, which is a coefficient, h, by particular investor. The equation says, if the standard deviation wanted is greater than standard deviation of the entire time-series by a factor of h, then it is indicated as 1 otherwise 0. The trading probability δ is then calculated by the ratio of this number over total number that the standard deviation can be calculated (total number of the population of the time-series).

In other words, we are looking at areas in the binomial distribution that are either too high or too low (far away from the mean).

7.4 Optimal Trading Price

How often do we have to trade? Each of us has a unique risk tolerance as well as expected return for our investment. Then each of us has a particular k we are using when making investment decisions. This is to say, we need to find the right amount of k around the price when the price-to-moving average is the lowest. The bigger k an investor has the earlier he needs to start buying.

The first goal is to find the optimal price-to-moving averages level. We need to take the first order derivative of price-to-moving averages and set the result to zero. We have the following theorem.

Theorem 7.4. *$\forall n$ and $\forall \triangle > 0$ no matter how small, when $\sum_{t=0}^{n} P_t = \sum_{t=0}^{n} P_{t+\triangle}$, we have a critical price to act on it.*

The interpretation is fairly intuitive. We look at a series of price and the price goes up or down. We calculate the average of prices in a certain period. Then we have a series of average prices. We look at the ratio of price over moving averages and we treat the ratio to be a new number series with some arbitrary mean. We calculate the sum of prices in the past certain of time nodes. The optimal price level occurs at a time when the sum of the averages at two time nodes does not change at all (or change tiny little). Intuitively speaking, this is a time when the price is going down too much and now it is starting to bounce back, causing the sum of the past prices to be relatively stable. For example, we calculate the sum of hundreds of prices in the past. If the price keeps going down, we are supposed to get a sum smaller and smaller, yet one day the sum stays the same all of the sudden. Then this price can be critical to buy and vice versa. The complete proof is shown in the Appendix 4.1.

In practice, we will probably find some price areas as critical price level. That is to say, we would not find perfect equivalent sums of prices in the data. We would probably see a range of prices that satisfy the condition. Then we can apply trading frequency δ. The bigger the δ is the bigger the time node is. Therefore, a consistent investor who is true to his own preference would start to buy earlier and vice versa (sell earlier). The smaller the δ is the smaller the time nodes is. A consistent investor would buy later and vice versa.

Remark 7.5. In the industry, people sometimes look at moving averages and exponential moving averages. Some even look at Bolinger Bands or Relative Strength Index to determine whether the market is overbought or oversold. The idea is similar as the the proof indicates.

One can always observe a series of price actions heading one direction until the momentum dies out and price direction reverses.

7.5 Optimal Trading Frequency

Now we have a systematic way to describe our trading frequency and the understanding of an ideal price to pull the trigger. The next question is very intuitive. How do we make sure that our trading frequency matches the occurrences of "good" prices?

The idea is the following. We can observe prices going up or down from the dataset. From the price, we calculate the standard deviation (by taking first order derivatives). Each investor has some arbitrary coefficient, h, to trade, which is multiplied to the standard deviation. For example, for a mean standard deviation $\bar{\sigma}$, an institutional investor would be heavier and trade less and he could have a coefficient of 2. That is, he would not buy/sell unless the security is at a price level that gives him standard deviation of twice more than the mean standard deviation $2 \times \bar{\sigma}$. As an opposite example, an individual trader would be lighter and trade more and he could have a coefficient of 1.6. That is, if the price gives him a standard deviation of 1.6 higher or lower than the mean, $1.6 \times \bar{\sigma}$, he would pull the trigger to buy/sell. How do we know a person is being consistent and rational with his ideal trading frequency?

The answer is to let the trading frequency matches the probability of the appearance of the optimal trading price, which leads to the following theorem.

Theorem 7.6. *A trader has optimum at $\sigma_{t,n} = \sigma_{t+\triangle,n}$ and he should set his trading coefficient h to be greater than $\frac{\sigma'_{t,n}}{\sigma_{t',n'}}$.*

This theorem is to show the audience a risk can be a good indicator of your trading frequency coefficient. A trader can observe price action and calculate some standard deviation, $\sigma_{t,n}$, at the beginning. As time moves on he observes more and more prices; and he can calculate an updated standard deviation, $\sigma'_{t,n}$. The theorem says one should trade only when he observes an extremely large updated standard deviation. In fact, he wants to observe a standard deviation that is greater than the standard deviation calculated by all the dataset in the past. Reader can see the proof in Appendix.

Remark 7.7. Majority of hedge funds and trading company conduct research on large data set. One week or even one more month of data would not affect the overall standard deviation by a lot. One needs to keep in mind that this approach may seem to have little difference day by day in practice.

The observed new standard deviation is a indicator that the risk is changing at that time node in the market. This can only happen with a lot of volume going on at that time node in the industry. This is usually due to some large counterparty unwinding his or her positions. This is usually a signal to get in or out of positions. This activity usually comes with significant volume.

7.6 Empirical Data

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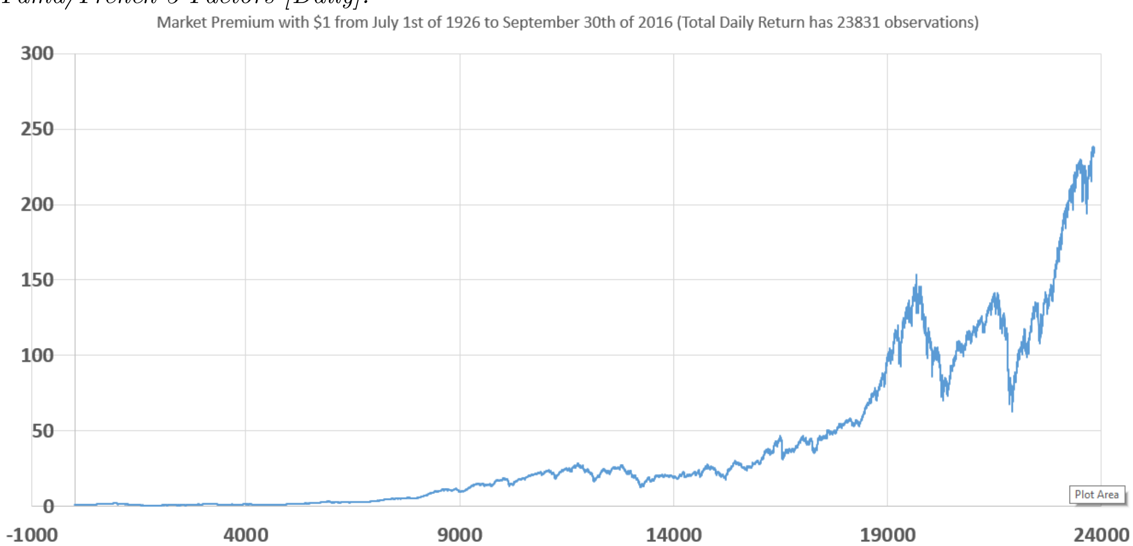
Now let us deviate from theoretical model and look at the data. We take daily returns of market premium from July 1st of 1926 to September 30th of 2016 as our data set. Starting from \$1 on the first date of the data set, we plot dollar value based on accumulated daily market premium, which is shown in Figure 5, by the formula

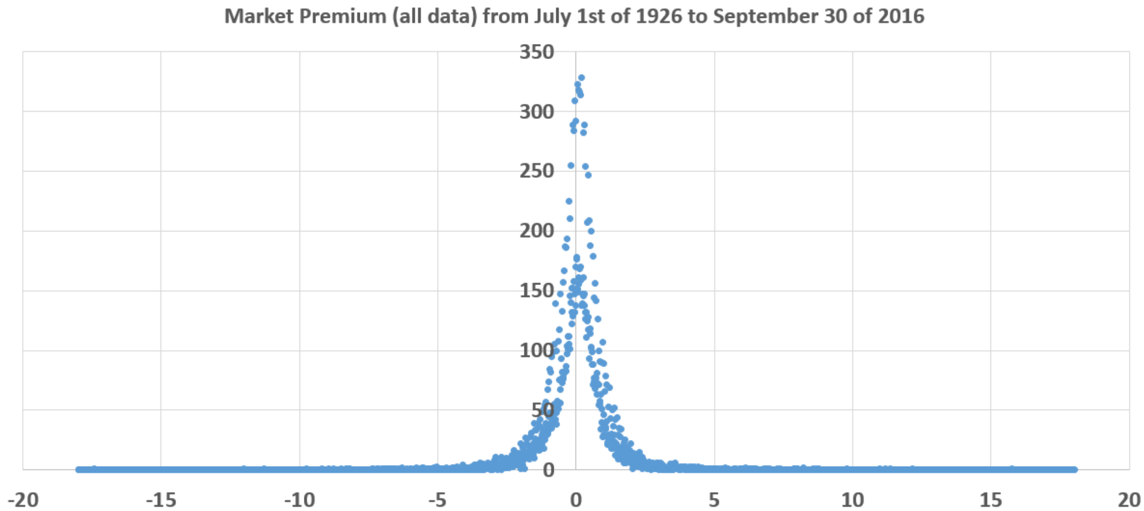
$$P_t = \$1(1 + r_t)(1 + r_{t-1}) \dots (1 + r_1) = \prod_{i=1}^{n}(1 + r_i).$$

Figure 15: The graph shows Market Premium with \$1 starting from July 1st of 1926 to September 30 of 2016 based on 23,831 observations (daily return). *Source: From Ken French Data Library, Fama/French 3 Factors [Daily].*



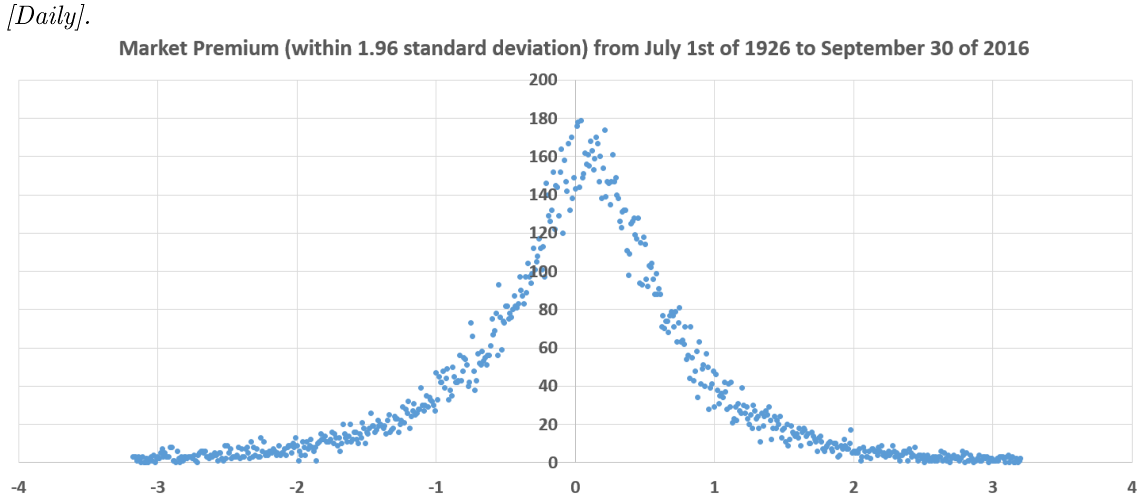
Next, Figure 6 presents market premium from Ken French Data Library. We plot all available daily return of market premium. We observe that the market premium follow normal distribution and that the 1.96 standard deviation is at 3.02%. That is, a $\pm 3.02\%$ return would cover approximately 97% of market premium from July 1st of 1926 to September 30 of 2016.

Figure 16: The graph shows Market Premium (all data) from July 1st of 1926 to September 30 of 2016. *Source: From Ken French Data Library, Fama/French 3 Factors [Daily].*



Then we take a closer look at the data only choosing the return within 1.96 standard deviation, which is presented in Figure 7.

Figure 17: The graph shows Market Premium (within 1.96 standard deviation) from July 1st of 1926 to September 30 of 2016. *Source: From Ken French Data Library, Fama/French 3 Factors [Daily].*



The data tells us that price follows random walk which confirms the definitions and theorems presented in previous subsections. We present a method to pick a critical price by using moving averages and investor trading frequency. We assume price follows a random walk and we assume, for now, sufficient liquidities for to make a market. We provide a model such that an investor can find a critical price to be local minimum or local maximum to act on.

We know price, P_t, follows martingale. Then $SMA_n = \frac{1}{n}\sum_{t=0}^{n} P_t$ must follow martingale as well. This implies that price-to-moving averages, $P_t - SMA_n$ (defined in Definition 6.2), are random as well. Based on normal distribution, we can extract statistically "normal" and "abnormal" data to identify what prices can we act on.

Below we present Figure 8 and Figure 9. Figure 8 presents price-to-moving averages on \$1 accumulated return based on market premium taking data only within 1.96 standard deviation. Figure 9 presents price-to-moving averages on \$1 accumulated return based on market premium taking data only within 1.96 standard deviation.

The plots present an enormous amount of volume happening on left-side of the tail. This phenomenon is obvious in Figure 8 data over 40- and 50-day SMA, and also in Figure 9 all the periods. In other words, the data failed to explain what happened outside of -1.96 standard deviation range.

Figure 18: The graph shows $1 of Market Premium and calculates price-to-moving averages over 10-, 20-, 30-, 40-, and 50-day period (taking data within 1.96 standard deviation with 0.01 increment) from July 1st of 1926 to September 30 of 2016. *Source: From Ken French Data Library, Fama/French 3 Factors [Daily].*

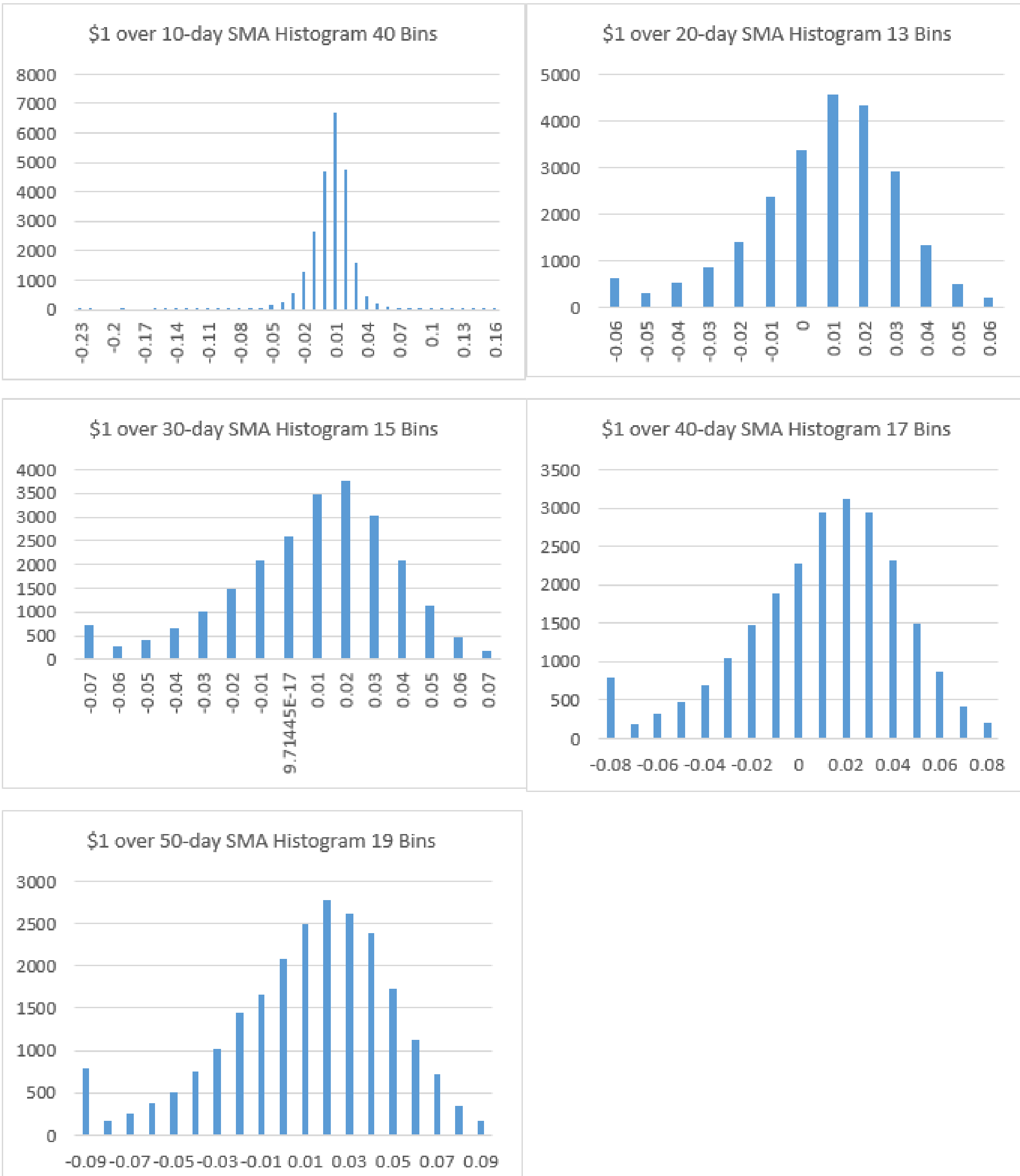
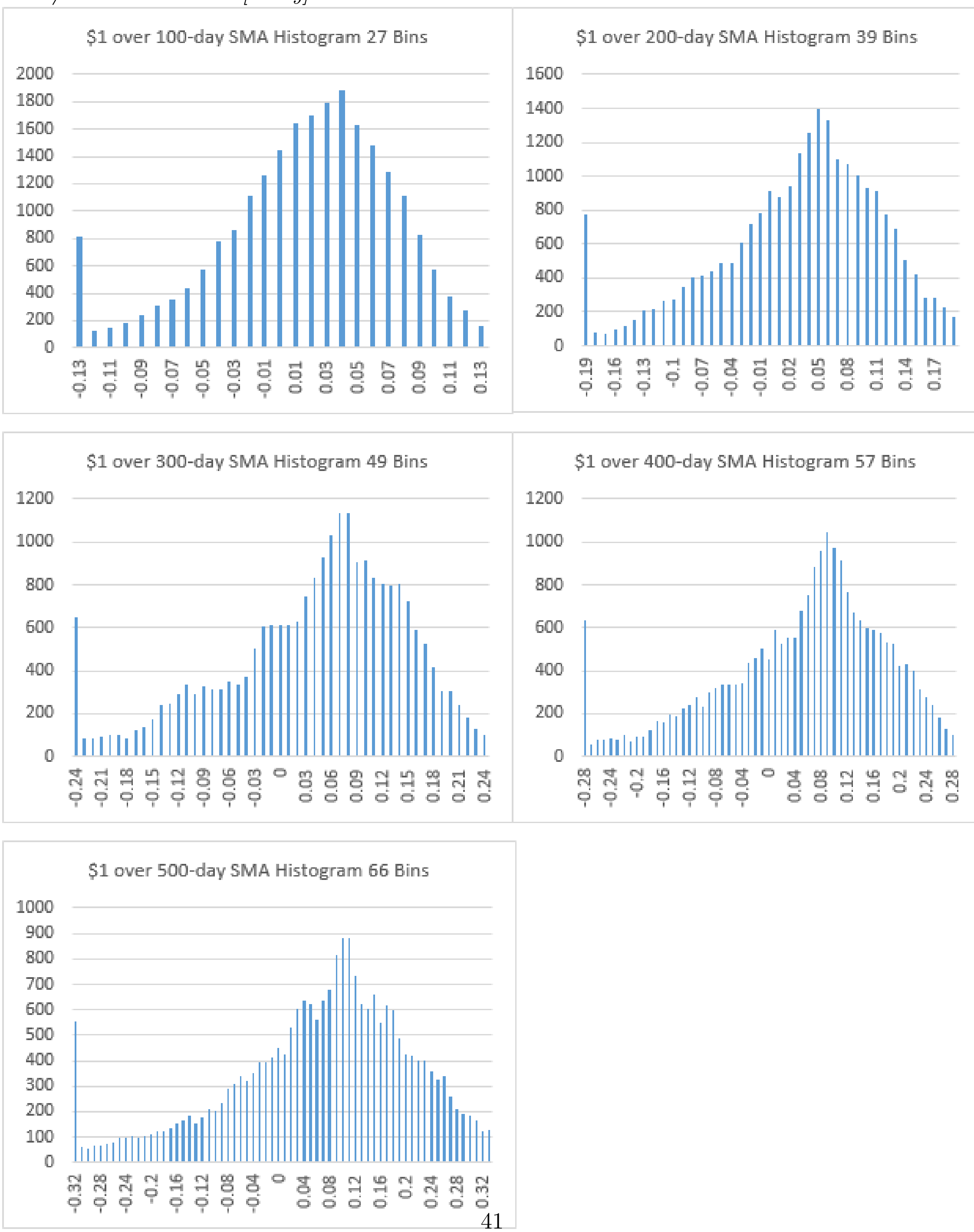


Figure 19: The graph shows $1 of Market Premium and calculates price-to-moving averages over 100-, 200-, 300-, 400-, and 500-day period (taking data within 1.96 standard deviation with 0.01 increment) from July 1st of 1926 to September 30 of 2016. *Source: From Ken French Data Library, Fama/French 3 Factors [Daily].*



In Table 6, we plot all the bins (with 0.01 margin increment) that we used to plot the normal distribution histogram. We calculated price-to-moving averages of 10-, 20-, 30-, 40-, 50-, 100-, 200-, 300-, 400-, and 500-day periods. Each period we computed 1.96 standard deviation and we used 0.01 margin increment within the range of 1.96 standard deviation. The table tells us a range of "normal" price activities based on different periods of moving averages. In practice, a trader can calculate price-to-moving averages and his "conviction" to buy should be consistent with the plot of Table 6. For example, let us say two scenarios market gives us a price-to-moving average of -0.13 and an alternative of -0.01. Holding all else equal, we argue a trader should feel more conviction to act on the price-to-moving average of -0.13 then of -0.01, because -0.13 is statistically less likely to happen than -0.01. One can also take a combination of multiple moving averages.

7.7 Discussion of Applications

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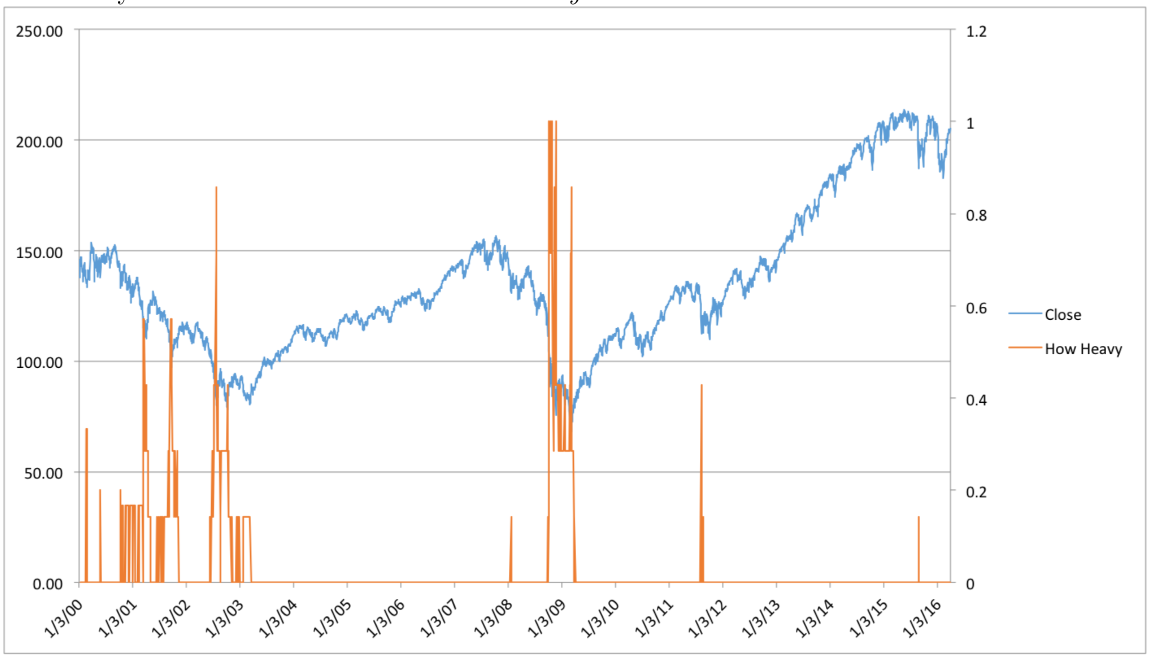
In practice, this model definitely can pick up some good price ranges. In Figure 5, we have shown the daily price plot for S&P 500 from January 2000 to January 2016 from Yahoo Data Library. We calculated the moving averages on 10-, 20-, 30-, 50-, 100-, 200-, and 300-day periods. For less than 10% trading frequency, we calculate the buy signal for each price-to-moving averages, which are presented as spikes on x-axis. The more price-to-moving averages based on different time periods satisfy the condition the higher the spikes are. The higher the spikes are the heavier an investor should buy. We observe that the spikes pretty much covered both financial crisis (2001 and 2008) and the model successfully tells us, by using only past data at each time node, that when to start buying and when to buy heavy.

Figure 20: The graph shows S&P 500 from January 2000 to January 2016. The model takes 10-, 20-, 30-, 50-, 100-, 200-, 300-day to be moving average periods and calculate the price-to-moving average ratio by investor trading frequency to be less than 10%. The spikes from x-axis are buy signal calculated based on critical price level. The higher the spikes are the heavier an investor should buy. *Source: From Yahoo Finance Library.*



We collect data and analyze. Based on some model, we generate a range of prices that we call data set. The newly observed price in the future could only fall in or out of the data set. If the new price falls in the prior distribution, we simply take original distribution. Inside the prior distribution, we call for *Alpha Protocol* if the new price hits ideal range. If the new price falls out of prior distribution, we discuss two scenarios (i.e. below or above prior distribution) and calculate posterior distribution. If the new price falls above prior distribution, the situation calls for *House Party Protocol*. If new price falls below prior distribution, the situation calls for *The Winter Contingency*.

Remark 7.8. Commodities trading can follow similar strategy. Commodities are difficult products to generate alpha. If desired, a trader can follow *House Party Protocol* and *The Winter Contingency* for trading strategies. Metal, especially Gold, in 2016 was a good example.

7.7.1 Alpha Protocal

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Alpha Protocal is an algorithmic action procedure designed with the foundational understanding of buying heavy at low price-to-moving averages with multi-period data. Given a certain amount of wealth, a money manager should follow the following steps to obtain full perspectives of the world market before he can make a rational decision.

The foundation, or the trigger, of this protocol is a series of low price activities that hit around two standard deviations in a certain security. In this procedure, low and high are relative terms. Referring to Table 6 Panel A, an observaton of $1 invested in market premium could be above or below 10-day moving averages with 0.0387% range. In other words, an interval of $[-0.04\%, 0.04\%]$ would cover approximately 98% of the price actions. Sometimes the price may drag some certain moving averages straight down for a long period of time. That is, there are occasions the price stays below 10-day moving averages for a long time. A solution is to look at longer time frame, that is, 20-day moving averages or above. However, data shows that 500-day moving averages with $\pm 2\sigma_{500-day}$ would cover approximately 97% of the data in the entire data set from July 1st of 1926 to September 30 of 2016. Let us assume an equity market capital of $20 trillion dollars. For an alpha buyer (perhaps a group of ETFs) investing in market premium, let us say for some reason he wants to invest $1 trillion into U.S. equity market. It would be ideal for him to buy the bottom 5% (=1/20) of the market. Thus, he should be thinking about entering this position when the price is about 30% below 500-day moving average (referring to Table 6 Panel A under the column of 500-day the third row). This idea leads to the following theory.

Given collected data on some security i at some time t. We compute moving averages over a period of time, k period into the past. Then we have $p_{i,t-k}$. We can compute price-to-moving averages (by Definition 6.2) to be $(p_{i,t-k})/[\frac{1}{n+1}\sum_{k=0}^{n} p_{i,t-k}]$ (since $k = 0$ counted as one extra time node). We say the ratio for n ($\forall n \in \mathbb{Z}$) period of time nodes into the past follows normal distribution. That is, $(p_{i,t-k})/[\frac{1}{n+1}\sum_{k=0}^{n} p_{i,t-k}] \sim \mathcal{N}(\mu_n, \sigma_n)$. Computing over multiple periods (multiple n's) at the same time, for $\tau_k = t - k$ ($\forall k \in \mathbb{Z}$), we have

$$\begin{bmatrix} \vdots \\ \mathcal{N}(\mu_{\tau_\mathbf{k}}, \sigma_{\tau_\mathbf{k}}) \\ \vdots \end{bmatrix} = \begin{bmatrix} \mathcal{N}(\mu_{\tau_\mathbf{1}}, \sigma_{\tau_\mathbf{1}}) \\ \mathcal{N}(\mu_{\tau_\mathbf{2}}, \sigma_{\tau_\mathbf{2}}) \\ \vdots \\ \mathcal{N}(\mu_{\tau_\mathbf{k}}, \sigma_{\tau_\mathbf{k}}) \end{bmatrix}.$$

With the above information in hand, we observe new price action (assume newly observed price fall in data set, see remark for otherwise), say $p_{i,t+1}$. Then, for some "buy" coefficient c_B such that $c_B \in \mathbb{R}^+$, we check the following

$$\mathbb{I}_{k,c_B} = \begin{cases} 1 \quad , & (p_{i,t-k})/[\frac{1}{n+1}\sum_{k=0}^{n} p_{i,t-k}] < \mu_{\tau_k} - c_B \times \sigma_{\tau_k} \\ 0 \quad , & otherwise \end{cases} ;$$

and also for some "sell" coefficient c_S such that $c_S \in \mathbb{R}^+$, we check the following

$$\mathbb{I}_{k,c_S} = \begin{cases} 1 \quad , & (p_{i,t-k})/[\frac{1}{n+1}\sum_{k=0}^{n} p_{i,t-k}] > \mu_{\tau_k} + c_S \times \sigma_{\tau_k} \\ 0 \quad , & otherwise \end{cases} .$$

A sum of "buy" indicators at time t will take the form of $\sum_{k=0}^{n} \mathbb{I}_{k,c_B}$. A sum of "sell" indicators at time t will take the form of $\sum_{k=0}^{n} \mathbb{I}_{k,c_S}$.

Remark 7.9. A rational money manager always observe new prices. If newly observed price falls above data range, this will call for *House Party Protocol*. That is, the price is so high that the security (or whatever product he is observing) is making historical highs. If newly observed price falls below data range, this will call for *The Winter Contingency*. That is, the price is so low that he has never seen it before.

Be cautious here. We need to observe macro money flow. Consider two parties, with i balances and j balances at a certain t with $T = t + \tau$ to be the total time up to now. Let $\phi_{ij}^{ks}(\tau)$ be the probability that the former and the latter hold k and s balances after the meeting, respectively. Then we have $\phi_{ij}^{ks}(\tau)$ to be in the interval $[0, 1]$ (closed since they do not have to trade or have to trade for sure), which implies that $\sum_{k\in\mathbb{K}}\sum_{s\in\mathbb{K}}\phi_{ij}^{ks}(\tau) = 1$. Given any possible distribution of trading probabilities, the distribution of balances at time t, which is $n(\tau) = \{n_k(\tau)\}_{n\in\mathbb{K}}$, evolves to be

$$\dot{n}_k(\tau) = f[n(\tau), \phi(\tau)], \ \forall k \in \mathbb{K} \tag{28}$$

where

$$f[\mathbf{n}(\tau), \phi(\tau)] \equiv \alpha n_k(\tau) \sum_{i\in\mathbb{K}}\sum_{j\in\mathbb{K}}\sum_{s\in\mathbb{K}} n_i(\tau)\phi_{ki}^{sj}(\tau) - \alpha \sum_{i\in\mathbb{K}}\sum_{j\in\mathbb{K}}\sum_{s\in\mathbb{K}} n_i(\tau)n_j(\tau)\phi_{ij}^{ks}(\tau), \tag{29}$$

which is the same as we discussed in previous section.

This distribution of balances introduced above can be used a major approach to identify large block trades in the market disregard what product the traders are trading. This is the most essential, if not the only essential factor, for a trade to be rational when newly observed price falls outside of the range of prior distribution.

From statistical point of view, suppose that the n random variables $X_1, ..., X_n$ form a random sample from a distribution for which the p.d.f./p.m.f. is $(\cdot|\theta)$. Suppose also that the value of the parameter θ is unknown and the prior p.d.f./p.m.f. of θ is $\zeta(\cdot)$. Then the posterior p.d.f./p.m.f. of θ is

$$\zeta(\theta|\mathbf{x}) = \frac{f(x_1|\theta)\dots f(x_n|\theta)\zeta(\theta)}{g_n(\mathbf{x})}, \forall\theta \in \Omega,$$

where g_n is the marginal joint p.d.f./p.m.f. of $X_1, ..., X_n$.

7.7.2 House Party Protocal

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When we observe new price that falls above prior distribution, $\mathcal{N}(\mu_{\tau_\mathbf{k}}, \sigma_{\tau_\mathbf{k}})$, we input this as a data point into our data set and compute again the posterior distribution, $\mathcal{N}'(\mu'_{\tau_\mathbf{k}}, \sigma'_{\tau_\mathbf{k}}|x_{new})$. Then we compare μ_{τ_k} and μ'_{τ_k} to see if the new data point pushes up the mean. Moreover, we check σ_{τ_k} and σ'_{τ_k} to see how if this increases the variance.

For a particular security i, one can observe currently traded price to be very close to historical high. In other words, he is observing a $p_{i,t}$ such that $p_{i,t+1} \geq p_{i,t} \in \mathcal{N}(\mu_{\tau_k}, \sigma_{\tau_k})$. At this point in which a *House Party Protocol* is triggered, a rational buyer, if he wants to enter this market, should observe the nearest bid. Given a set of bidding strategy $b_i : [0, \omega] \to \mathbb{R}_+$. The definition of auction theory defines the winning bid to be the highest bid or the second to highest bid in order to get the stock. To enter the market, a trader can use market order or limit order (putting as inside big to get the stock). A market order would guarantee him to own the stocks but with the highest price. A limit order would guarantee him the price he puts down but with no promise of getting the stocks. Observing the next to best bids, a rational trader would know to lean on the liquidity below him. In other words, for player one $i = 1$ with bidding strategy b_1, he faces two possibilities in *House Party Protocol*: 1) the stock breaks historical high and moves higher, or 2) the stock touches historical high and bounces back. If player one is in the first scenario, he is happy and simply needs to execute his original strategy. If play one is in the second scenario, he needs to exit his position. A rational trader should always be aware of risk, which is largely affected by liquidity below his price level in this scenario. He observes the next to best bid, say player two $i = 2$ with bidding strategy b_2. Player One, disregard what is the probability he thinks this trade would be profitable, would never ever take a position larger than the liquidity provider's position at his exit price. Since this trade is a small risk small reward strategy, we assume a trader would want to exit his position at the next to best bid. In other words, his strategy should suffice $b_1 \leq b_2$. Player One can be have different risk tolerance, so we can restate his strategy to be a set $B_1 = \{b_1 : b_1 \in [0, \omega] \to \mathbb{R} \textit{ and } b_1 \leq b_2\}$. In other words, $\limsup_{\omega} B_1 = b_2$.

To have good control on the risk, he needs not only to make sure the exit price has no price slippage but also to make sure the entry price has no price slippage. Follow the same logic, a rational trader should be aware of the asks (seller of the stocks). Given a set of asking strategy $s_i : [0, \gamma] \to \mathbb{R}_+$. For Player One (the same Player One mentioned above) who wants to enter his position with no price slippage, he should always observe the national best offer (i.e. ask), say s_i. In this case, his bidding strategy b_1 should never be larger than what the national best ask can offer for him to not cause any price slippage at the entry. Putting everything together, we have $B_1 = \{b_1 : b_1 \in [0, \omega] \to \mathbb{R} \textit{ and } b_1 \leq \min(s_i, b_2)\}$. After entry and well planned exit, he can execute his plans (or whatever strategy he uses) to sell or hold his stocks towards the upside.

Such procedure is important because a rational trader would not want to chase high into a position while the same time there is a trigger of "sell" signal of *Alpha Protocol*. In the cases that the newly observed price is not that high (on the street people say "extended"), it is reasonable to take a trade to the upside, in which the trader wants to buy high sell higher. The weight of this strategy should not be more than the risk tolerance of a trader, which is a factor unquantifiable and solely based on the trader himself.

This price action described in *House Party Protocol* is consistent with the §*5 Asset Pricing*, in which I discussed the concept of "greed". When price is a lot higher than enterprise value, price tend to extend and fall sharply eventually. This is, in relative concept, a greedy action caused by irrational measure of risk and reward ratio.

Remark 7.10. In practice, traders use all sorts of signals. *Resistance* and *support* are price levels that traders look at. They are formed by previously occurred daily-, weekly-, monthly-chart highs/lows. Most notable ones are called bull-flag, a chart pattern defined by a resistance and a support. Some traders also refer to pennant, flag, wedge, inside-bar, and other patterns. There is no scientific evidence to show the reliance of these patterns. Once in a while, there are a few experienced traders who often talk about these patterns. In reality, experienced traders would use multiple layers of other information that happen at the same time to establish psychological conviction, a method to trade with a healthy mentality (mostly a strategy consistent with short- and long-run reversals, some say "red-dog reversal").

Remark 7.11. The *Volcker Rule* refers to §619 [44] (12 U.S.C. §1851) part of the Dodd-Frank Wall Street Reform and Consumer Protection Act, originally proposed by American economist and former United States Federal Reserve Chairman Paul Volcker to restrict United States banks from making certain kinds of speculative investments that do not benefit their customers. This is an action prevents less greed in *House Party Protocol*.

Remark 7.12. I personally do not take major positions based on *House Party Protocol*. That is not to say that one should never calls in this strategy. Instead, one should trade with great caution, small tier size, and usually accompanied with an already-successful *Alpha Protocol* executed.

In the cases that the position does not break new higher prices than current high prices, traders, given the condition is a small risk high reward strategy, would want to get out of the position. This largely depends on how much liquidity is provided at the price the trader wants to get out, which goes back to the concept of balance distribution between two block traders (discussed above, i.e. balance distribution $f[\mathbf{n}(\tau), \phi(\tau)]$). For two parties of a trade

i and j at the exit price, we observe $f[\mathbf{n}(\tau), \phi(\tau)] \equiv \alpha n_k(\tau) \sum_{i\in\mathbb{K}} \sum_{j\in\mathbb{K}} \sum_{s\in\mathbb{K}} n_i(\tau)\phi_{ki}^{sj}(\tau) - \alpha \sum_{i\in\mathbb{K}} \sum_{j\in\mathbb{K}} \sum_{s\in\mathbb{K}} n_i(\tau)n_j(\tau)\phi_{ij}^{ks}(\tau)$. Let us say i is the buy-side and j is the sell-side. It does not make sense to take a position that is too big for party i to accept. If this is the case, there will be a slippage, an exit that is completed with one order broken into multiple smaller orders with different prices (usually lower prices). This is will cause transaction imbalance between two parties i and j. A series of such activities (in 2008 an entire country's traders were doing the same thing) would trigger *The Winter Contingency.*

7.7.3 The Winter Contingency

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You never know if there will be a sufficiently larger trader who is willing to buy all of your shares at the price that you are able to tolerate. That is, given some range of observed prices, a trader has some model or strategy that he follows and he has an exit plan. However, what if the market does not give him his ideal exit plan. Or worse yet, what if the market gives him an exit plan that is too low for him to even realize what had just happened? This would trigger *The Winter Contingency.*

In situations like this, we observe a new price that falls below prior distribution, $\mathcal{N}(\mu_{\tau_\mathbf{k}}, \sigma_{\tau_\mathbf{k}})$, we input this newly observed price as a data point into our data set and compute again the posterior distribution, $\mathcal{N}'(\mu'_{\tau_\mathbf{k}}, \sigma'_{\tau_\mathbf{k}}|x_{new})$. In most cases, if a trader follows the first two protocols, it would be very unlikely for him to trigger *The Winter Contingency.* However, if he does not, he would have been more easily to find himself in a "Winter Contingency" situation. When this happens, he should immediately compare μ_{τ_k} and μ'_{τ_k} to see if the new data point actually drags down the mean. Moreover, we check σ_{τ_k} and σ'_{τ_k} to see if this increases the variance. If yes, he should then check how much does this drag him down.

Given the occurrence of above situation and a trader's strategy, this trader would be facing two options: 1) exit the position, or 2) buy more. The first option is related to traders with sufficiently large risk that the price already hit his maximum level of risk tolerance). The second option would be related to traders with limited risk (or sufficiently small) risk that the price has not yet hit his maximum level of risk tolerance.

However, one needs to be aware the probability to survive this position in terms of time. Let Y denote survival time, and let $f_Y(\tau)$ be the probability density function for this situation. The cumulative distribution of Y is then $\int_0^\tau f_Y(t)dt$, which gives us the probability of failure in terms of time y. The survivor function is the complement of probability of failure. Thus, we have the following definition.

Definition 7.13. Let Y be survival time, $f_Y(\tau)$ be p.d.f., the c.d.f. of Y is

$$F_Y(\tau) = P(Y \leq y) = \int_0^\tau f_Y(t)dt,$$

and the *survival function* is defined as

$$S_Y(\tau) = P(Y > \tau) = 1 - F_Y(\tau).$$

Theorem 7.14. *Suppose in time τ we have survivor function $S_Y(\tau)$ and hazard function $h_Y(\tau) = \frac{f_Y(\tau)}{S_Y(\tau)}$. Then we have*

$$S_Y(\tau) = \exp[-H_Y(\tau)]$$

while

$$H_Y(\tau) = \int_0^y h_Y(t)dt.$$

to describe the relationship between survivor function and hazard function. In other words, the longer the time τ the greater the risk of failure.

The interpretation of this theorem is the following. For a trader who is holding a position at a price below his exit price, the risk of failure is exponentially increasing. Once can use *hazard function* to describe the risk in respect of time. The proof is presented in Appendix.

Remark 7.15. One should notice that one data point of observation does not mean anything. In other words, majority of *The Winter Contingency* is based on "fear" (even an event like that happened in 2008 was largely due to fear).

In worst cases, if a trader truly wants to get out of one position, a rational action is to sell little by little instead of selling everything at the same time. An immediately sell off would cause more fear in the market, which would attracts two types of people. Short sellers would be attracted to come into the market to take over the liquidities on the bid side to prevent you from getting out of position without slippage. Buyers would cancel bids because they do not know how many shares are you selling. Rational buyers would be patient and wait for a trigger of *Alpha Protocol*, causing you to lose more money or wait for an even longer time to unwind your position.

Remark 7.16. (Uptick Rule). The uptick rule is a trading restriction that states that short selling a stock is only allowed on an uptick. For the rule to be satisfied, the short must be either at a price above the last traded price of the security, or at the last traded price when the most recent movement between traded prices was upward (i.e. the security has traded below the last-traded price more recently than above that price). The rule went into effect in 1938 and was removed when Rule 201 Regulation SHO became effective in 2007. In 2009, the reintroduction of the uptick rule was widely debated, and proposals for a form of its reintroduction by the SEC went into a public comment period on April 8 of 2009 [45] [46]. This act would certain protects traders who let themselves fall into the fearful situations. However, this merely stops the acceleration of falling price and would not recover the market.

Ever after, The NYSE, Nasdaq, and SEC continue to develop and test circuit breakers that will act as a safety net during times of exorbitant price changes in the market as a whole or in the price of individual securities.

Government can help us to a certain amount, but what if this is a type of situation Government cannot help us. Or worse yet, what if this is a two-trader market? This can be a case when a private equity manager bought an entire company only afterwards he realized that this company is not doing a good job and would need to file Chapter 11 or Chapter 12. Then you need to find more player in the game, which leads to §*8 Game Theory*.

8 §Game Theory§

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Game Theory in stock market is about supply and demand. There are machines assisting traders, but we still need to understand what game we are in and what are our least worst options. This section describes what we should be aware of in terms of executions.

8.1 Hide Your Order

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First round, it is optimal for bidders to shade their bids to account for the option value of participating in the subsequent second round [42]. Bidders with a higher valuation also have a higher option value. Therefore, they shade their bids in the first round by a greater amount than do bidders with a lower valuation. As the auction proceeds, the number of bidders decreases. In time, there will be a decreasing amount of liquidities provider and there would simply be no point to trade.

Over the sequence of auctions, the number of objects decreases as well. The first fact has a negative effect on the competition for an object and the second has a positive effect. Both effects cancel out and prices follow a martingale. As a result, all gains to waiting are arbitraged away and the expected prices in both rounds are the same. The latter result also holds for sequential auctions of more than two objects and does not depend on whether the auction is English or Dutch. In United States, Security Act 1934 defines secondary market to follow Dutch Auction, and hence rational traders feel indifferent about the effects above. However, this effects still play important roles in other markets outside of United States.

8.2 No Player, No Trade, No Profit

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This subsection discusses the following phenomenon: no player, no trade, no profit. The above theory that the two effects would cancel out receive some empirical evidence against them. Archenfelter (1989) [4] found a mild price decrease in sequential auctions of identical units of wine. McAfee and Vincent (1993) [37] also present empirical evidence on sequential wine auctions. They found that, on average, the second unit of wine was sold at a price 1.4 percent lower then the price of the first unit.

This theory can be applied in stock market yet it has its exceptions. Consider a unit of stock for any security. The bidders form a pool of demands, which are represented by price and shares. In sequential auctions, demand tends to fade away. This is because every time a trade is executed on the floor there will be some units of demands getting diggested by the market. In other words, holding all else equal, the demand will temporarily be lowered as soon as a trade is executed and printed on the tape. The same logic follows for the 2^{nd} trade, 3^{rd} trade, ... and so on. If there is new buying power coming in to the auction market, it is really difficult for a trade to work in a trader's direction.

9 §Conclusion§

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This document takes readers through macroeonomics, fundamentals, asset pricing, statistics in trading, and game theory. The author provides a way of filtering information. Market is like an invisible hand pushing price up and down. Facing this random walk, we can follow a series of protocols to direct our investment decisions. In the end of the day, it is you who press the button. A successful money manager needs to be imaginative; and he is certainly required to study and to practice, years of it!

10 §Tables§

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Table 1. The table uses *compustat* universe from 1950 to 2014 monthly data as sample. The table is sorted by the percentile of how much Enterprise Value (EV) changed in the past 12 months over Total Assets, value weighted from small to large (1-10) portfolios by NYSE breaks on FF-3. The entries without brackets are coefficients and the entries with brackets are t-statistics. The figure presents the access returns of each portfolios.

	XRet	Alpha	MKT	SMB	HML
1	0.367	-0.233	1.138	0.160	-0.288
	[1.78]	[-3.34]	[67.43]	[6.50]	[-10.88]
2	0.644	0.071	1.082	0.200	-0.293
	[3.25]	[1.09]	[68.17]	[8.61]	[-11.78]
3	0.547	0.022	1.039	-0.053	-0.221
	[3.04]	[0.35]	[68.92]	[-2.43]	[-9.35]
4	0.561	0.039	0.949	-0.070	-0.070
	[3.51]	[0.69]	[69.28]	[-3.47]	[-3.27]
5	0.693	0.122	0.982	-0.129	0.049
	[4.32]	[2.14]	[71.08]	[-6.36]	[2.26]
6	0.683	0.160	0.916	-0.062	-0.014
	[4.48]	[2.93]	[69.50]	[-3.19]	[-0.68]
7	0.679	0.086	0.927	-0.105	0.194
	[4.47]	[1.40]	[62.85]	[-4.88]	[8.38]
8	0.616	0.007	0.938	-0.042	0.187
	[3.95]	[0.11]	[60.61]	[-1.86]	[7.69]
9	0.839	0.153	0.993	0.153	0.213
	[4.92]	[2.23]	[59.97]	[6.33]	[8.18]
10	0.869	0.095	1.081	0.297	0.241
	[4.49]	[1.15]	[54.04]	[10.17]	[7.67]
L/S	0.502	0.328	-0.057	0.137	0.529
	[4.36]	[3.13]	[-2.24]	[3.71]	[13.31]

Table 2. This is a summary table for all L/S strategy portfolio from Table (1) - (16), using the *compustat* universe from 1950 to 2014 monthly data as sample. The table is sorted by how much Enterprise Value (EV) in Total Assets, change on Enterprise Value (EV) over Total Assets, and Enterprise Value (EV) over Total Assets, value weighted from small to large (1-5, 1-10) portfolios by NYSE breaks on FF-3. The first row represents the excess returns, alpha, market premium, small-minus-big, and high-minus-low. The following rows are separated by the sorts accordingly. The entries without brackets are coefficients and the entries with brackets are t-statistics. SR_1 refers to the Sharpe Ratio for the series of portfolios equally weighted. SR_2 refers to the Sharpe Ratio for the series of portfolios value weighted.

	XRet	Alpha	MKT	SMB	HML	XRet	Alpha	MKT	SMB	HML	SR_1	SR_2
$\omega_i = EV_i/TA_i$												
	LS in quintiles					LS in deciles						
L/S	0.169	0.300	0.140	0.071	-0.654	0.103	0.289	0.135	0.164	-0.849	0.1865	0.0900
	[1.49]	[3.41]	[6.58]	[2.29]	[-19.54]	[0.72]	[2.65]	[5.13]	[4.24]	[-20.46]		
$\phi_i = \triangle EV_i/TA_i = EV_i - lag(EV_i, 12)/TA_i$												
	LS in quintiles					LS in deciles						
L/S	0.333	0.201	-0.082	0.011	0.516	0.502	0.328	-0.057	0.137	0.529	0.4299	0.5511
	[3.40]	[2.43]	[-4.12]	[0.37]	[16.45]	[4.36]	[3.13]	[-2.24]	[3.71]	[13.31]		
Replica of traditional FF-3 sort by market value												
	LS in quintiles					LS in deciles						
L/S	0.446	-0.164	0.064	0.373	1.225	0.461	-0.282	0.074	0.583	1.431	0.3066	0.2529
	[2.88]	[-3.10]	[6.09]	[21.76]	[80.78]	[2.38]	[-3.41]	[4.52]	[21.69]	[60.16]		

Table 3. This is a summary table of the alphas from each L/S strategy for all of the sorts of Enterprise Value (EV). Panel A is the sort of the first order of Enterprise Value (EV) during the past t-period. Panel B is the sort of the change of Enterprise Value over Market Equity (ME) during the past t-period. Panel C is the sort of the second order of Enterprise Value (EV) during the past t-period.

Panel A	XRet	t-stat	Alpha	t-stat	+/- L or S
Sort by $\delta EV_{i,t} = (EV_{i,t} - lag(EV_{i,t}, t))/lag(EV_{i,t}, t)$					
12-month	0.172	1.06	0.394	2.48	t-(t-1)
24-month	0.093	0.56	-0.169	-1.10	(t-1)-t
36-month	0.266	1.64	-0.001	-0.01	(t-1)-t
48-month	0.272	1.75	0.010	0.01	(t-1)-t
60-month	0.233	1.52	-1.040	-0.32	(t-1)-t
Panel B	XRet	t-stat	Alpha	t-stat	+/- L or S
Sort by $\triangle EV_{i,t}/ME_{i,t} = (EV_{i,t} - lag(EV_{i,t}, t))/ME_{i,t}$					
12-month	0.026	0.17	-0.169	-1.10	(t-1)-t
24-month	0.232	1.49	-0.004	-0.02	(t-1)-t
36-month	0.406	2.61	0.142	1.02	(t-1)-t
48-month	0.343	2.31	0.146	1.13	(t-1)-t
60-month	0.383	2.73	0.207	1.70	(t-1)-t
Panel C	XRet	t-stat	Alpha	t-stat	+/- L or S
Sort by $\delta(\delta EV_{i,t})_{i,t} = (\delta EV_{i,t} - lag(\delta EV_{i,t}, t))/lag(\delta EV_{i,t}, t)$					
12-month	0.155	1.78	0.107	1.23	(t-1)-t
24-month	0.107	1.14	0.065	0.70	(t-1)-t
36-month	0.073	0.78	-0.006	-0.06	(t-1)-t
48-month	0.129	1.42	0.037	0.40	(t-1)-t
60-month	0.102	0.93	-0.008	-0.07	(t-1)-t

Table 4. This is a summary table of the alphas from each L/S strategy for all of the sorts of moving averages. There are four panels in this table, D, E, F, and G. each panel is sorted by different different definition derived from moving averages. Panel D is sort by the SMA of cross-sectional stock returns. Panel E is sort by the EMA of cross-sectional stock returns . Panel F is sorted by the difference between cross-sectional stock returns and SMA. Lastly, Panel G is sorted by the difference between cross-sectional stock returns and EMA.

Panel D	XRet	t-stat	Alpha	t-stat	+/- L or S
Sort by $SMA(\delta P_{i,t})_n = \frac{1}{n}\sum_{j=1}^{n}(\delta P_{i,t})_j$,					
10-month	0.285	1.72	0.179	1.08	(t-1)-t
20-month	0.285	1.72	0.179	1.08	(t-1)-t
30-month	0.285	1.72	0.179	1.08	(t-1)-t
40-month	0.285	1.72	0.179	1.08	(t-1)-t
50-month	0.285	1.72	0.179	1.08	(t-1)-t
Panel E	XRet	t-stat	Alpha	t-stat	+/- L or S
Sort by $EMA(\delta P_{i,t})_n = (\delta P_{i,t})\frac{1}{(n+1)} + \frac{1}{n}\sum_{j=1}^{n}(\delta P_{i,t})_j(1-\frac{1}{(n+1)})$					
10-month	0.528	3.62	0.461	3.15	(t-1)-t
20-month	0.495	3.39	0.419	2.88	(t-1)-t
30-month	0.497	3.41	0.424	2.91	(t-1)-t
40-month	0.344	2.37	0.253	1.75	(t-1)-t
50-month	0.514	3.53	0.448	3.08	(t-1)-t
Panel F	XRet	t-stat	Alpha	t-stat	+/- L or S
Sort by $\delta P_{i,t} - SMA(\delta P_{i,t})_n$					
10-month	0.484	3.36	0.420	2.92	(t-1)-t
20-month	0.473	3.28	0.398	2.77	(t-1)-t
30-month	0.475	3.29	0.399	2.77	(t-1)-t
40-month	0.179	1.30	0.346	2.57	(t-1)-t
50-month	0.483	3.36	0.411	2.86	(t-1)-t
Panel G	XRet	t-stat	Alpha	t-stat	+/- L or S
Sort by $\delta P_{i,t} - EMA(\delta P_{i,t})_n$					
10-month	0.484	3.36	0.420	2.92	(t-1)-t
20-month	0.473	3.28	0.398	2.77	(t-1)-t
30-month	0.475	3.29	0399	2.77	(t-1)-t
40-month	0.180	1.31	0.347	2.58	(t-1)-t
50-month	0.483	3.36	0.411	2.86	(t-1)-t

Table 5. This is a continued table from Table 3. The table summarizes of all the FF-3 factor coefficients for L/S replicas and corresponding t-statistics for all three panels. Panel A is the sort of the change of Enterprise Value (EV) over Enterprise Value (EV) during the past t-period. Panel B is the sort of the change of Enterprise Value over Market Equity (ME) during the past t-period. Panel C is the sort of the second order of Enterprise Value (EV) during the past t-period.

Panel H	Alpha	MKT	SMB	HML	Panel I	Alpha	MKT	SMB	HML
12-month	0.394	-0.018	-0.191	-0.511		-0.169	0.023	0.265	0.385
	[2.48]	[-0.46]	[-3.40]	[-8.47]		[-1.10]	[0.62]	[4.90]	[6.64]
24-month	-0.169	-0.082	0.407	0.673		-0.004	-0.058	0.320	0.601
	[-1.10]	[-2.23]	[7.53]	[11.63]		[-0.02]	[-1.65]	[6.28]	[10.99]
36-month	-0.001	-0.121	0.419	0.696		0.142	-0.107	0.393	0.679
	[-0.01]	[-3.49]	[8.27]	[12.78]		[1.02]	[-3.21]	[8.06]	[12.95]
48-month	0.001	-0.163	0.386	0.810		0.146	-0.186	0.347	0.654
	[0.01]	[-5.42]	[8.60]	[16.75]		[1.13]	[-5.98]	[7.70]	[13.43]
60-month	-0.004	-0.148	0.238	0.844		0.207	-0.176	0.241	0.609
	[-0.32]	[-4.96]	[5.53]	[18.21]		[1.70]	[-5.97]	[5.67]	[13.31]

Panel J	Alpha	MKT	SMB	HML
12-month	0.107	-0.044	-0.18	-0.118
	[1.23]	[-2.10]	[-5.89]	[-3.59]
24-month	0.065	-0.075	0.189	0.125
	[0.70]	[-3.36]	[5.85]	[3.57]
36-month	-0.006	-0.024	0.132	0.174
	[-0.06]	[-1.06]	[4.09]	[5.01]
48-month	0.037	0.044	0.127	0.120
	[0.40]	[2.02]	[4.07]	[3.56]
60-month	-0.008	-0.002	0.144	0.211
	[-0.07]	[-0.07]	[3.88]	[5.25]

Table 6 (Panel A). This is a summary table showing all negative values for all the bins with 0.01 margin increment. We calculate 1.96 standard deviation for 10-, 20-, 30-, 40-, 50-, 100-, 200-, 300-, 400-, and 500-day price-to-moving averages based on market premium extracted from Ken French Data Library, July 1st of 1926 to September 30 of 2016. *Source: Ken French Data Library, Fama/French 3 Factors [Daily].* Note that Panel A shows all the negative bins and Panel B shows all the positive bins.

10-	20-	30-	40-	50-	100-	200-	300-	400-	500-
0.0387	0.0563	0.0704	0.0824	0.0931	0.1343	0.1900	0.2373	0.2782	0.3118
									-0.32
									-0.31
									-0.3
									-0.29
								-0.28	-0.28
								-0.27	-0.27
								-0.26	-0.26
								-0.25	-0.25
							-0.24	-0.24	-0.24
							-0.23	-0.23	-0.23
							-0.22	-0.22	-0.22
							-0.21	-0.21	-0.21
							-0.2	-0.2	-0.2
						-0.19	-0.19	-0.19	-0.19
						-0.18	-0.18	-0.18	-0.18
						-0.17	-0.17	-0.17	-0.17
						-0.16	-0.16	-0.16	-0.16
						-0.15	-0.15	-0.15	-0.15
						-0.14	-0.14	-0.14	-0.14
					-0.13	-0.13	-0.13	-0.13	-0.13
					-0.12	-0.12	-0.12	-0.12	-0.12
					-0.11	-0.11	-0.11	-0.11	-0.11
					-0.1	-0.1	-0.1	-0.1	-0.1
				-0.09	-0.09	-0.09	-0.09	-0.09	-0.09
			-0.08	-0.08	-0.08	-0.08	-0.08	-0.08	-0.08
		-0.07	-0.07	-0.07	-0.07	-0.07	-0.07	-0.07	-0.07
	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06	-0.06
	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05
-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04
-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03
-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02
-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
0	0	0	0	0	0	0	0	0	0

Table 6 (Panel B). This is a summary table showing all positive values for all the bins with 0.01 margin increment. We calculate 1.96 standard deviation for 10-, 20-, 30-, 40-, 50-, 100-, 200-, 300-, 400-, and 500-day price-to-moving averages based on market premium extracted from Ken French Data Library, July 1st of 1926 to September 30 of 2016. *Source: Ken French Data Library, Fama/French 3 Factors [Daily].* Note that Panel A shows all the negative bins and Panel B shows all the positive bins.

10-	20-	30-	40-	50-	100-	200-	300-	400-	500-
0.0387	0.0563	0.0704	0.0824	0.0931	0.1343	0.1900	0.2373	0.2782	0.3118
0	0	0	0	0	0	0	0	0	0
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
		0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
			0.08	0.08	0.08	0.08	0.08	0.08	0.08
				0.09	0.09	0.09	0.09	0.09	0.09
					0.1	0.1	0.1	0.1	0.1
					0.11	0.11	0.11	0.11	0.11
					0.12	0.12	0.12	0.12	0.12
					0.13	0.13	0.13	0.13	0.13
						0.14	0.14	0.14	0.14
						0.15	0.15	0.15	0.15
						0.16	0.16	0.16	0.16
						0.17	0.17	0.17	0.17
						0.18	0.18	0.18	0.18
						0.19	0.19	0.19	0.19
							0.2	0.2	0.2
							0.21	0.21	0.21
							0.22	0.22	0.22
							0.23	0.23	0.23
							0.24	0.24	0.24
								0.25	0.25
								0.26	0.26
								0.27	0.27
								0.28	0.28
									0.29
									0.3
									0.31
									0.32
									0.33

11 §Appendix§

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11.1 Proof of Theorem 2.1

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Proof: Price P_t, a discrete-time stochastic variable, is adapted to $\mathbb{F}$, that is, P_t is F_t-measurable. Moreover, $\mathbb{E}|P_t| < \infty$. Last, $E(P_{t+1}|F_t) = P_t$, that is, the expected price of the next time node is independent as the expected price of the current time node. We have seen empirical evidence from Malkiel (2007) [36].

We can also check for first order derivative of price $r_t = \frac{\partial P}{\partial t}$. We see that r_t is F_t-measurable. Also $\mathbb{E}|r_t| < \infty$. Last, $E(r_{t+1}|F_t) = r_t$.

Q.E.D.

11.2 Proof of Theorem 2.2

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Proof: We start with (ii) and then prove (i). Assume that for a given t and s, we can write $\exists k > 0, t = s + k$. Since $\mathbb{F} = \{F_n, n = 0, 1, ...\}$ is a filtration, we have the following by tower property

$$\begin{aligned}
\mathbb{E}(r_t|F_s) &= \mathbb{E}(\mathbb{E}(r_t|F_{t-1})|F_s) = \mathbb{E}(r_{t-1}|F_s) \\
&= \mathbb{E}(\mathbb{E}(r_{t-1}|F_{t-2})|F_s) = \mathbb{E}(r_{t-2}|F_s) \\
&= ... \\
&= \mathbb{E}(\mathbb{E}(r_{t-(k-2)}|F_{t-(t-1)})|F_s) = \mathbb{E}(r_{t-(k-1)}|F_s) \\
&= \mathbb{E}(r_{s+1}|F_s) = r_s
\end{aligned}$$

Hence, this completes proof for (ii). For (i), we take $s = 0$ and (ii) gives us $\mathbb{E}(r_t|F_0) = r_0$. Then the expectations will be

$$\mathbb{E}(r_t) = \mathbb{E}(\mathbb{E}(r_t|F_0)) = \mathbb{E}(r_0)$$

which completes the proof.

Q.E.D.

11.3 Proof of Theorem 2.4

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Proof: We have price-to-moving average to be the following. For n periods, we have

$$P_t - SMA_n = \frac{P_t}{\frac{1}{n}\sum_{t=0}^{n} P_t} - 1$$

We want to know what time is the price the lowest/highest. Thus, we want to take derivative in respect with time t. Then set the first order derivative to be zero and we have

$$\frac{\partial(P_t - SMA_n)}{\partial t} = lim_{\triangle \to 0} \frac{(\frac{P_{t+\triangle}}{\frac{1}{n}\sum_{t=0}^{n} P_{t+\triangle}} - 1) - (\frac{P_t}{\frac{1}{n}\sum_{t=0}^{n} P_t} - 1)}{t} = 0$$

$$
\begin{aligned}
\Rightarrow \quad & lim_{\triangle\to 0}\frac{\frac{P_{t+\triangle}}{\frac{1}{n}\sum_{t=0}^{n}P_{t+\triangle}}-\frac{P_t}{\frac{1}{n}\sum_{t=0}^{n}P_t}}{t} &=& \quad 0\\
\Rightarrow \quad & lim_{\triangle\to 0}\left(\frac{1}{t}\right)\left(\frac{P_{t+\triangle}}{\frac{1}{n}\sum_{t=0}^{n}P_{t+\triangle}}-\frac{P_t}{\frac{1}{n}\sum_{t=0}^{n}P_t}\right) &=& \quad 0\\
\Rightarrow \quad & lim_{\triangle\to 0}\left(\frac{n}{t}\right)\left(\frac{P_{t+\triangle}}{\sum_{t=0}^{n}P_{t+\triangle}}-\frac{P_t}{\sum_{t=0}^{n}P_t}\right) &=& \quad 0\\
\Rightarrow \quad & lim_{\triangle\to 0}\left(\frac{n}{t}\right)\Big((P_{t+\triangle})\times(\textstyle\sum_{t=0}^{n}P_t)-(P_t)\times(\sum_{t=0}^{n}P_{t+\triangle})\Big) & &\\
& \times(\textstyle\sum_{t=0}^{n}P_t\times\sum_{t=0}^{n}P_{t+\triangle})^{-1} & &\\
& &=& \quad 0
\end{aligned}
$$

Then we need

$$\Rightarrow\left((P_{t+\triangle})\times(\sum_{t=0}^{n}P_t)-(P_t)\times(\sum_{t=0}^{n}P_{t+\triangle})\right)=0$$

Since $P_{t+\triangle}=P_t$ when $\triangle\to 0$ (the price this second is very close to the price next second), we can factor out one of them

$$\Rightarrow(P_{t+\triangle})\times\left((\sum_{t=0}^{n}P_t)-(\sum_{t=0}^{n}P_{t+\triangle})\right)=0$$

$$\Rightarrow(\sum_{t=0}^{n}P_t)-(\sum_{t=0}^{n}P_{t+\triangle})=0$$

Hence, when $(\sum_{t=0}^{n}P_t)=(\sum_{t=0}^{n}P_{t+\triangle})$, we have the optimal price in the market that has the first order derivative to be zero (or approximately zero), which shows that this price is a local minimum/maximum (a critical price or an actionable price).

Q.E.D.

11.4 Proof of Theorem 3.5

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Proof: At the optimal trading price, we have $(\sum_{t=0}^{n}P_t)=(\sum_{t=0}^{n}P_{t+\triangle})$. For each time node t and $t+\triangle$, we have

$$\sigma_{t,n}=\sqrt{\frac{1}{n}\sum_{i=1}^{n}((p_t-SMA_i)-\frac{1}{n}\sum_{i=1}^{n}(p_t-SMA_i))^2}$$

and

$$\sigma_{t+\triangle,n}=\sqrt{\frac{1}{n}\sum_{i=1}^{n}((p_{t+\triangle}-SMA_i)-\frac{1}{n}\sum_{i=1}^{n}(p_{t+\triangle}-SMA_i))^2}$$

Since $(\sum_{t=0}^{n}P_t)=(\sum_{t=0}^{n}P_{t+\triangle})$, then we have $\sigma_{t,n}=\sigma_{t',n'}$. Then for a particular time node t and the next unit of time node $t+\triangle$,

$$\delta_t=Pr(\sigma'_{t,n}<h\sigma_{t,n})=\frac{\sum_{(t',n')}\mathbb{I}_{(t',n')}}{\sum_{(t,n)}\mathbb{I}_{(t,n)}},$$

we have $\mathbb{I}_{(t',n')}$ remains the same while $\mathbb{I}_{(t,n)}$ counts one more event. That is, we have $h'\sigma_{t',n'}=h'\sigma_{t'+\triangle,n'},\forall h'\in\mathbb{R}^+$. Note that here h' is a different investor coefficient as the definition. Recall the definition that

$$\mathbb{I}_{(t',n')} = \begin{cases} 1, \ when \ h\sigma_{t',n'} > \sigma'_{t,n} \\ 0, \ when \ h\sigma_{t',n'} \leq \sigma'_{t,n} \end{cases}, \ (sign \ changes \ for \ sell \ side)$$

which has an initial investor coefficient h. The initial coefficient is defined in the model. However, as time moves on and investor observes different return and standard deviation, he will have another coefficient, h'. You simply want to adjust trading frequency h' to match the condition $(\sum_{t=0}^{n} P_t) = (\sum_{t=0}^{n} P_{t+\triangle})$.

That is, a trader has original coefficient h, and he wants to set h to h'. When the indicator is 1, he trades. The indicator is 1 if standard deviation gets above or below a certain coefficient h multiplied to the standard deviation. Now the optimal standard deviation is when $\sigma_{t,n} = \sigma_{t+\triangle,n}$. Then we have the following

$$h\sigma_{t',n'} > \sigma'_{t,n}$$

$$\Rightarrow h > \frac{\sigma'_{t,n}}{\sigma_{t',n'}},$$

yet since optimum occurs at $\sigma_{t,n} = \sigma_{t+\triangle,n}$, then

$$\begin{array}{rcl} \Rightarrow \frac{\sigma_{t,n}}{dt} & = & \lim_{\triangle\to\infty} \frac{\sigma_{t+\triangle,n} - \sigma_{t,n}}{\triangle} \\ \Rightarrow \frac{\sigma_{t,n}}{dt} & = & 0 \ as \ \triangle \to \infty. \end{array}$$

Hence, we conclude a trader has optimum at $\sigma_{t,n} = \sigma_{t+\triangle,n}$ and he should set his trading coefficient h to be greater than $\frac{\sigma'_{t,n}}{\sigma_{t',n'}}$.

Q.E.D.

11.5 Proof of Theorem 7.14

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Proof: By definition, we have

$$f_Y(\tau) = \lim_{\partial\tau\to 0} \frac{F_Y(\tau+\partial\tau) - F_Y(\tau)}{\partial\tau},$$

we have the hazard function to be the following

$$\begin{array}{rcl} h_Y(\tau) & = & \lim_{\partial y\to 0} \frac{F_Y(\tau+\partial\tau)-F_Y(\tau)}{\partial\tau S_Y(\tau)} \\ & = & \lim_{\partial\tau\to 0} \frac{P(\tau<Y\leq\tau+\partial\tau)}{\partial\tau S_Y(\tau)} \\ & = & \lim_{\partial\tau\to 0} \frac{P(\tau<Y\leq\tau+\partial\tau|Y>\tau)}{\partial\tau} \end{array}$$

Recall the definition of hazard function, $h_Y(\tau)\frac{f_Y(\tau)}{S_Y(\tau)}$ and we can relate hazard function and survivor function, which gives us

$$\begin{array}{rcl} h_Y(\tau) & = & \frac{f_Y(\tau)}{S_Y(\tau)} \\ & = & \frac{f_Y(\tau)}{1-F_Y(\tau)} \\ & = & -\frac{\partial}{\partial\tau}\ln[1-F_Y(\tau)] \\ & = & -\frac{\partial}{\partial\tau}\ln[S_Y(\tau)] \end{array}$$

Therefore, we have

$$S_Y(\tau) = \exp[-H_Y(\tau)]$$

while

$$H_Y(\tau) = \int_0^y h_Y(t)dt.$$

Q.E.D.

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